Answer to Question #88246,Physics / Molecular Physics | Thermodynamics

An engine using 1 mol of an ideal gas initially at 21.7 L and 326 K performs a cycle consisting of four steps:

1) an isothermal expansion at 326 K from 21.7L to 42.1 L ;

2) cooling at constant volume to 186 K ;

3) an isothermal compression to its original volume of 21.7L; and

4) heating at constant volume to its original temperature of 326K .

Find its efficiency. Assume that the heat capacity is 21 J/K and the universal gas constant is 0.08206 L · atm/mol/K = 8.314 J/mol/K.

Answer in units of %.

Solution:n=1, V_1=21.7L

i) At T1=326K, Volume changes from V_1=21.7L to V_2=42.1L (Say the path A to B)

It is a constant temperature process where the gases are expanded from volume V_A to V_B. Thus is a isothermal expansion. In this case heat is absorbed by the gas.

Thus work is done by the gas when heat is added.

ii) Cooling at constant volume (Say path B to C). Temperature decreases from T_1=326K to T_2=186K.

In this case no any heat is added and since it is constant volume process, the work is done is zero

iii) At temperature T_2=186K, the volume is decreases from V_C=42.1L to V_D=21.70L(say path C to D. It is a constant temperature process where the gas is compressed from V_2 to V_1.

Since the volume is decreases so work is done on the gas but no any heat is added.

iv) Heating at constant volume to its original temperature of 326 K (Say path D to A).
Volume is constant, but temperature is increases from T_2=186K to T_1=326K
Work is done on the gas and heat is rejected.

Thus efficiency of the engine= Total work done/Net heat added

Path AB,

Work done=$nRT_{hot}\ln(V_B/V_A)$

Path BC,

Work done=0

Path CD,

Work done=$nRT_{cold}\ln(V_D/V_C)$

Path DA,

Work done =0

Heat absorbed,

Path AB,

Q_1=W1= $nRT_{hot}\ln(V_B/V_A)$

Path DA,

$Q_4=C_v$ deltaT=$Cv(T_{hot}-T_{cold})$

Efficiency=

$$\left\{nRT_{hot}\ln\left({V_B}/{V_A}\right)+nRT_{cold}\ln\left({V_D}/{V_C}\right)\right\}\Big/\left\{nRT_{hot}\ln\left({V_B}/{V_A}\right)+C_v\left(T_{hot}-T_{cold}\right)\right\}$$

$$\left\{T_{hot}\ln\left({V_B}/{V_A}\right)+T_{cold}\ln\left({V_D}/{V_C}\right)\right\}\Big/\left\{T_{hot}\ln\left({V_B}/{V_A}\right)+{C_v\left(T_{hot}-T_{cold}\right)}/{nR}\right\}$$

$$\left\{326\ln\left({42.1}/{21.7}\right)+186\ln\left({21.7}/{42.1}\right)\right\}\Big/\left\{326\ln\left({42.1}/{21.7}\right)+{21(326-186)}/{1\times 8.314}\right\}$$

={326x0.66-186x0.66}/{326×0.66+21x16}

=92.4/551.16

=16.76%