Answer on question #86452, Physics / Mechanics | Relativity

Given:

(1) $v_p = c * \lambda^{\frac{1}{2}}$

Formulae:

(2) $v_p = \frac{\omega}{k}$ Where ω *is angular frequency and* k *is wavenumber*$(\frac{2\pi}{\lambda})$

(3) $v_g = \frac{d\omega}{dk}$

Solution:

From 1 and 2

$$\frac{\omega}{k} = c * \lambda^{\frac{1}{2}}$$

$$\omega = c * \lambda^{\frac{1}{2}} * k$$

$$\omega = c * (2\pi/k)^{\frac{1}{2}} * k$$

$$\omega = c * (2\pi)^{\frac{1}{2}} * k^{\frac{3}{2}} \qquad (4)$$

Taking derivative of equation 4 and substitute in equation 3, we get

Group velocity = $\frac{3}{2} * \sqrt{2\pi} * c * \sqrt{k}$

Group velocity = $\frac{3}{2} * c * \frac{2\pi}{\lambda} * \sqrt{\lambda}$

Group velocity = $\frac{3\pi}{\lambda} * phase\ velocity$

Since $v_p = c * \lambda^{\frac{1}{2}}$ therefore $\lambda = \left(\frac{v_p}{c}\right)^2$

On substituting, we get

Group velocity = $3\pi c^2 * (phase\ velocity\ )^{-1}$