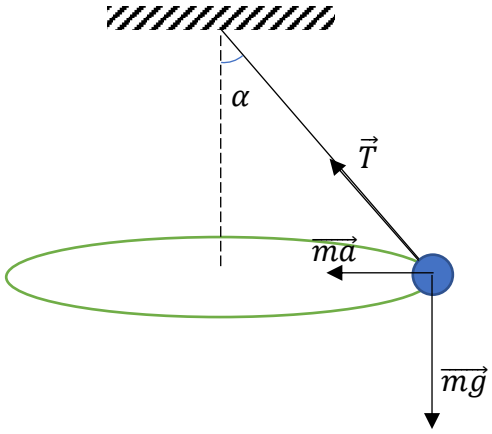
Answer on question # 85866 - Physics / Mechanics | Relativity

A 10 kg object is spun around in a circle by a rope horizontally above the ground. The length of the rope is currently 5 m. How much more or less force is needed to maintain a velocity of 12 m/s if the rope is extended by 3 m?

Solution

1) Figure



2) 2nd Newton's law

$$\vec{T} + \overrightarrow{mg} = \overrightarrow{ma}$$

3) Acceleration

$$a = \frac{mv^2}{R}$$

Where

$$R = lsin(\alpha)$$

4) Since, from the vectors having a right triangle, then the force T

$$T = \sqrt{(mg)^2 + (ma)^2}$$

$$T = \sqrt{(mg)^2 + \left(\frac{mv^2}{lsin(\alpha)}\right)^2}$$

$$T = m\sqrt{g^2 + \left(\frac{v^2}{lsin(\alpha)}\right)^2}$$

5) Define α when $l = 5\ m$ and force in this case
   a. Define α

$$tan(\alpha) = \frac{v^2}{gR} = \frac{v^2}{glsin(\alpha)}$$

$$\frac{v^2}{gl} = \frac{1 - \cos^2(\alpha)}{\cos(\alpha)}$$

   i. variable replacement

$$\cos(\alpha) = x$$

Then

$$x^2 + \frac{v^2}{gl}x - 1 = 0$$
$$x^2 + \frac{144}{10 * 5}x - 1 = 0$$
$$x^2 + 2{,}88x - 1 = 0$$

Solution for this

$$x_{1,2} = \frac{-2{,}88 \pm \sqrt{2{,}88^2 + 4}}{2}$$
$$x_1 = 0{,}31; x_2 = -3{,}2$$

$\cos(\alpha)$ –can not be > 1

in this way

$$\alpha = \arccos(0{,}31) = 72°$$

b. Force T

$$T = m\sqrt{g^2 + \left(\frac{v^2}{lsin(\alpha)}\right)^2}$$
$$T = 10\sqrt{10^2 + \left(\frac{144}{5 * sin(72)}\right)^2} = 320\ (N)$$

6) Define α when *l = 8 m* and force in this case

a. Define α

$$tan(\alpha) = \frac{v^2}{gR} = \frac{v^2}{glsin(\alpha)}$$
$$\frac{v^2}{gl} = \frac{1 - \cos^2(\alpha)}{\cos(\alpha)}$$

i. variable replacement

$$\cos(\alpha) = x$$

Then

$$x^2 + \frac{v^2}{gl}x - 1 = 0$$
$$x^2 + \frac{144}{10 * 8}x - 1 = 0$$
$$x^2 + 1{,}8x - 1 = 0$$

Solution for this

$$x_{1,2} = \frac{-1{,}8 \pm \sqrt{1{,}8^2 + 4}}{2}$$
$$x_1 = 0{,}44; x_2 = -2{,}24$$

$\cos(\alpha)$ –can not be > 1

in this way

$$\alpha = \arccos(0{,}44) = 64°$$

b. Force T

$$T = m\sqrt{g^2 + \left(\frac{v^2}{lsin(\alpha)}\right)^2}$$

$$T = 10\sqrt{10^2 + \left(\frac{144}{8 * sin(64)}\right)^2} = 224\ (N)$$

Answer:

1) When length 5 m, force 320 N

2) When length 8 m, force 224 N

It needs less force