The density of a ball is (300 ± 8) kg·m^{-3}. The ball is placed on a metre rule to find its diameter. One end of a diameter is opposite the 35 cm mark and the other end is at 78 cm and the error in each of these measurements is ± 1 cm. Find the mass of the ball.

Given:

$\rho = (300 \pm 8) kg/m^3$

$x_1 = (35 \pm 1)\ cm$

$x_2 = (78 \pm 1)\ cm$

$m-?$

Solution.

Let's find the mass of the ball:

$$m = \rho V = \frac{4}{3}\pi\rho R^3 = \frac{4}{3}\pi\rho\left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi\rho(x_2 - x_1)^3$$

$$m = \frac{1}{6} \cdot 3.14 \cdot 300 \cdot (0.75 - 0.35) = 62.8\ (kg)$$

Let's calculate the absolute error:

$$\Delta m \approx dm = \left|\frac{\partial m}{\partial \rho}\right| |\Delta\rho| + \left|\frac{\partial m}{\partial x_1}\right| |\Delta x_1| + \left|\frac{\partial m}{\partial x_2}\right| |\Delta x_2| =$$

$$= \left|\frac{\partial}{\partial \rho}\left(\frac{1}{6}\pi\rho(x_2 - x_1)^3\right)\right| |\Delta\rho| + \left|\frac{\partial}{\partial x_1}\left(\frac{1}{6}\pi\rho(x_2 - x_1)^3\right)\right| |\Delta x_1| + \left|\frac{\partial}{\partial x_2}\left(\frac{1}{6}\pi\rho(x_2 - x_1)^3\right)\right| |\Delta x_2| =$$

$$= \frac{1}{6}\pi(x_2 - x_1)^3\Delta\rho + \frac{1}{6}\pi\rho \cdot 3(x_2 - x_1)^2\Delta x_1 + \frac{1}{6}\pi\rho \cdot 3(x_2 - x_1)^2\Delta x_2 =$$

$$= \frac{1}{6}\pi(x_2 - x_1)^3\Delta\rho + \frac{1}{6}\pi\rho \cdot 3(x_2 - x_1)^2(\Delta x_1 + \Delta x_2)$$

$$\varepsilon = \frac{\Delta m}{m} = \frac{\frac{1}{6}\pi(x_2 - x_1)^3\Delta\rho + \frac{1}{6}\pi\rho \cdot 3(x_2 - x_1)^2(\Delta x_1 + \Delta x_2)}{\frac{1}{6}\pi\rho(x_2 - x_1)^3} = \frac{\Delta\rho}{\rho} + 3\frac{\Delta x_1 + \Delta x_2}{x_2 - x_1}$$

Relative error:

$$\varepsilon = \frac{8}{300} + 3\frac{0.01 + 0.01}{0.75 - 0.35} \approx 0.18$$

Than absolute error:

$$\Delta m = m\varepsilon = 62.8 \cdot 0.18 = 11.3 \approx 12\ (kg)$$

Finally the mass of the ball:

$$m = (62.8 \pm 12)\ kg \approx (63 \pm 12)\ kg$$

Answer: $m = (63 \pm 12\ )\ kg$