Answer on Question #85462, Physics / Other

If vector $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$ and $\mathbf{C} = C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}$.

Show that the scalar triple product of the vector is $[A_x\ A_y\ A_z][B_x\ B_y\ B_z\ ][C_x\ C_y\ C_z]$.

Solution:

The vector product by definition

$$[\mathbf{B}\times\mathbf{C}] = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \mathbf{i}\left(B_yC_z - B_zC_y\right) + \mathbf{j}(B_zC_x - B_xC_z) + \mathbf{k}(B_xC_y - B_yC_x)$$

The scalar product by definition

$$(\mathbf{A}\cdot\mathbf{B}) = A_xB_x + A_yB_y + A_zB_z$$

So, for the triple product we obtain

$$(\mathbf{A}\cdot[\mathbf{B}\times\mathbf{C}]) = A_x\left(B_yC_z - B_zC_y\right) + A_y(B_zC_x - B_xC_z) + A_z\left(B_xC_y - B_yC_x\right)$$

$$= \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$