

Question # 85455, Physics / Mechanics | Relativity

Task: The equation of a stationary wave on a string fixed at both ends is given by $y(x, t) = 2 \sin(\pi x) \cos(100\pi t)$, where x and y are measured in metre and t in second. Calculate the amplitude, wavelength and frequency of component waves whose superposition generated this stationary wave. Also write the equations of component waves.

Solution:

$$y(x, t) = 2 \sin(\pi x) \cos(100\pi t) = \sin(\pi x + 100\pi t) + \sin(\pi x - 100\pi t) = \sin(\pi x + 100\pi t) + \sin(100\pi t - \pi x + \pi)$$

General formula for a standing wave: $y(x, t) = A \sin(\omega t - kx + \phi)$.

We have $y(x, t) = y_1(x, t) + y_2(x, t)$.

Consider first component $y_1(x, t) = \sin(\pi x + 100\pi t)$:

amplitude is $A_1 = 1$ m

angular frequency is $\omega_1 = 100\pi$ rad/s

frequency is $f_1 = \omega_1/2\pi = 50$ Hz

wave number is $k_1 = -\pi$ rad/m

velocity is $v_1 = \omega_1/k_1 = -100$ m/s

(velocity is negative because the wave is travelling in the negative x direction)

wavelength is $\lambda_1 = |v_1|/f_1 = 2$ m

Consider second component $y_2(x, t) = \sin(100\pi t - \pi x + \pi)$:

amplitude is $A_2 = 1$ m

angular frequency is $\omega_2 = 100\pi$ rad/s

frequency is $f_2 = \omega_2/2\pi = 50$ Hz

wave number is $k_2 = \pi$ rad/m

velocity is $v_2 = \omega_2/k_2 = 100$ m/s

wavelength is $\lambda_2 = |v_2|/f_2 = 2$ m