Answer on Question #85321- Physics / Electromagnetism

Question: *Using Maxwell's equation in free space,derive the wave equations for the z-component of electric field*
vector

Answer:

The Maxwell's equations in free space have the following form (Gauss units are used):

$$\begin{aligned} \nabla\cdot\vec{E} &= \rho, \\ \nabla\cdot\vec{H} &= 0, \\ \nabla\times\vec{E} &= -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \\ \nabla\times\vec{H} &= \frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\vec{j}. \end{aligned} \qquad (1)$$

Calculating the curl from the left side of the third equation in (1), one can derive:

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \Delta\vec{E} = \nabla\rho - \Delta\vec{E} \qquad (2)$$

Taking into account that operators ∇ and ∂/∂t commute, one can derive the corresponding expression after taking the curl from the right side of the same equation:

$$\nabla\times\left(-\frac{1}{c}\frac{\partial\vec{H}}{\partial t}\right) = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\vec{j}\right) = -\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial\vec{j}}{\partial t} \qquad (3)$$

Combining the expressions together, we obtain:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \nabla\rho + \frac{4\pi}{c^2}\frac{\partial\vec{j}}{\partial t} \qquad (4)$$

Finally, projecting this equation on the z-axis, one can derive:

$$\Delta E_z - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = \frac{\partial\rho}{\partial z} + \frac{4\pi}{c^2}\frac{\partial j_z}{\partial t} \qquad (5)$$