Answer on Question #84504 - Physics - Mechanics | Relativity

Problem

the tape in a video cassette has a total length 190 m and can play for 2.5 h. as the tape starts to play, the full reel has an outer radius of 36 mm and an inner radius of 14 mm. at some point during the play, both reels will have the same angular speed. what is the common angular speed? answer in units of rad/s

Solution.

$\boldsymbol{l = 190\ m}$
$\boldsymbol{t = 2.5\ h = 9000\ s}$
$\boldsymbol{R = 36\ mm = 0.36\ m}$
$\boldsymbol{r = 14mm = 0.14\ m}$
$\boldsymbol{w}-?$

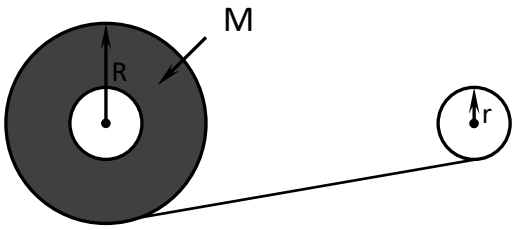

Fig 1

According to definition of angular velocity we will have:

$$w = \frac{v}{r}, \tag{1}$$

where v – velocity of circling point , r – radius of circle.
Velocity of circling point equally velocity of motion of the film, $v = const$:

$$v = \frac{l}{t}.$$

When $r_1 = r_2$, reels will have the same angular velocity (fig. 2)

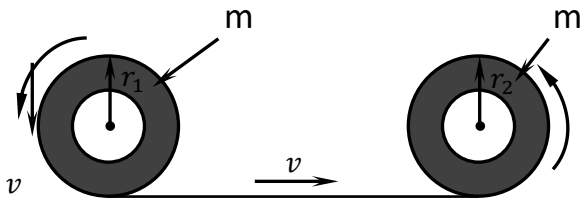

Fig 2

We can use the law of mass conservation and find r_1.
From Fig. 1:

$$M = \frac{V}{\rho} = \frac{Sh}{\rho} = \frac{(\pi R^2 - \pi r^2)h}{\rho} = \frac{\pi h}{\rho}(R^2 - r^2).$$

From Fig. 2:

$$M = 2m = 2\,\frac{V_1}{\rho} = 2\frac{S_1 h}{\rho} = 2\frac{(\pi r_1^2 - \pi r^2)h}{\rho} = 2\frac{\pi h}{\rho}(r_1^2 - r^2).$$

So,

$$\frac{\pi h}{\rho}(R^2 - r^2) = 2\frac{\pi h}{\rho}(r_1^2 - r^2)$$

$$R^2 - r^2 = 2(r_1^2 - r^2)$$

$$R^2 - r^2 = 2r_1^2 - 2r^2$$

$$2r_1^2 = R^2 - r^2 + 2r^2$$

$$2r_1^2 = R^2 + r^2$$

$$r_1^2 = \frac{R^2 + r^2}{2}$$

$$r_1 = \frac{\sqrt{R^2 + r^2}}{\sqrt{2}}.$$

From (1):

$$w = \frac{v}{r_1} = \frac{l}{tr_1} = \frac{l\sqrt{2}}{t\sqrt{R^2 + r^2}}$$

$$w = \frac{190\sqrt{2}}{9000\sqrt{0.36^2 + 0.14^2}} = 0.077 \text{ (rad/s)}$$

Answer:

$$w = 0.077 \text{ (rad/s)}$$