Answer on Question #81637 - Physics - Mechanics – Relativity

Determine the lateral deflection of a cannonball that is shot in London (51 degree 31' N) and flies for 25s at an average horizontal speed of 120m/s. What would be the lateral deflection in Murmansk (68 degree 52'N) and Nairobi (1 degree 18'S)?

Solution

- Radius of the Earth: $R = 6375000$ m;
- Length of a day: $T = 24 \cdot 3600$ s.

Imagine that you are sitting in a chair and observe the cannonball flight. If it is shot along longitudinal line, its horizontal rate is

$$L_{\text{h}} = v \cdot t = 120 \cdot 25 = 3000 \text{ m},$$

and its angular rate in ° is

$$\alpha = \frac{L_{\text{h}}}{R} \cdot \frac{180°}{\pi} = \frac{vt}{R} \cdot \frac{180°}{\pi}.$$

It was shot in London. This fact means that from latitude of London the cannonball flew to latitude α in °:

$$\varphi = \varphi_{\text{L}} - \alpha = \varphi_{\text{L}} - \frac{vt}{R} \cdot \frac{180°}{\pi}.$$

At this latitude the radius of a circle perpendicular to the Earth rotational axis is

$$r = R \cdot \cos\varphi = R\cos\left(\varphi_{\text{L}} - \frac{vt}{R}\right).$$

For these 25 seconds of flight the Earth turned anti-clockwise (northern hemisphere) for angle in rad

$$\beta = 2\pi\frac{t}{T}$$

and arc

$$D = \beta \cdot r.$$

Remember that you are still sitting in a chair doing calculations in London? For 25 seconds at your latitude the Earth did an arc (and the cannonball did this arc as well)

$$d = \beta \cdot R\cos\varphi_{\text{L}}.$$

Thus, the lateral deflection for the cannonball shot from London will be

$$D_L = d - D = \beta R(\cos\varphi - \cos\varphi_{\text{L}}) =$$

$$= 2\pi\frac{t}{T} \cdot R\left[\cos\left(\varphi_{\text{L}} - \frac{vt}{R} \cdot \frac{180°}{\pi}\right) - \cos\varphi_{\text{L}}\right] =$$

$$= 2\pi\frac{25}{24 \cdot 3600} \cdot 6375000\left[\cos\left(51°31' - \frac{120 \cdot 25}{6375000} \cdot \frac{180°}{\pi}\right) - \cos 51°31'\right] = 4.3 \text{ m}.$$

From Murmansk:

$$D_M = 2\pi\frac{25}{24\cdot 3600}\cdot 6375000\left[\cos\left(68°52' - \frac{120\cdot 25}{6375000}\cdot\frac{180°}{\pi}\right) - \cos 68°52'\right] = 5.1 \text{ m}.$$

From Nairobi:

$$D_N = 2\pi\frac{25}{24\cdot 3600}\cdot 6375000\left[\cos\left(1°18' - \frac{120\cdot 25}{6375000}\cdot\frac{180°}{\pi}\right) - \cos 1°18'\right] = 0.12 \text{ m}.$$

Answer

$D_L = 4.3$ m, $D_M = 5.1$ m, $D_N = 0.12$ m.