Question #81570, Physics / Other

Imagine a flying super-dog that is able to accelerate horizontally and vertically. You are going to play fetch with this flying dog. Imagine throwing a rock up and forwards so that it follows the usual parabolic path away from you. As you throw the rock, the dog is waiting a few meters behind you. What is the acceleration vector for the dog if it flies in a straight line to catch the rock just as it passes the highest point on the parabola?

Solution

1) Consider the rock as projectile. At the highest point on the parabola:

$$x = v\cos\alpha\, t$$

$$y = v\sin\alpha\, t - \frac{gt^2}{2}.$$

$$t = \frac{2v\sin\alpha}{g}$$

2)

$$x = \frac{a_x t^2}{2}$$

$$y = \frac{a_y t^2}{2} - \frac{gt^2}{2}.$$

Thus,

$$\frac{a_x t^2}{2} = v\cos\alpha\, t$$

$$a_x = \frac{2v\cos\alpha}{t} = \frac{2v\cos\alpha}{\frac{2v\sin\alpha}{g}} = g\cot\alpha$$

$$v\sin\alpha\, t - \frac{gt^2}{2} = \frac{a_y t^2}{2} - \frac{gt^2}{2}.$$

$$a_y = \frac{2v\sin\alpha}{t} = \frac{2v\sin\alpha}{\frac{2v\sin\alpha}{g}} = g$$

Thus, the acceleration vector for the dog if it flies in a straight line to catch the rock just as it passes the highest point on the parabola is

$$\boldsymbol{a} = (g\cot\alpha\,, g)$$