Answer on Question #81251 Physics / Mechanics

The speed of sound v in a medium depends on its wavelength λ, the young modulus E, and the density ρ, of the medium. Use the method of dimensional analysis to derive a formula for the speed of sound in a medium.

Solution:

Let

$$v = \lambda^{\alpha} E^{\beta} \rho^{\gamma}$$

Since

$$[\lambda] = [\mathrm{m}] = \mathrm{L}$$
$$[E] = [\mathrm{Pa}] = \mathrm{ML^{-1}T^{-2}}$$
$$[\rho] = [\mathrm{kg/m^3}] = \mathrm{ML^{-3}}$$
$$[v] = [\mathrm{m/s}] = \mathrm{LT^{-1}}$$

We get

$$\mathrm{LT^{-1}} = \mathrm{L}^{\alpha}(\mathrm{ML^{-1}T^{-2}})^{\beta}(\mathrm{ML^{-3}})^{\gamma}$$

Or

$$\begin{cases} \alpha - \beta - 3\gamma = 1 \\ \beta + \gamma = 0 \\ -2\beta = -1 \end{cases}$$

The solution of this system of linear equations

$$\begin{cases} \alpha = 0 \\ \beta = 1/2 \\ \gamma = -1/2 \end{cases}$$

Therefore

$$v = \lambda^0 E^{1/2} \rho^{-1/2} = \sqrt{\frac{E}{\rho}}$$

Answer: $v = \sqrt{\frac{E}{\rho}}$