Question:

A girl is sitting with her dog at the left end of a boat of length 10.0m. The mass of the girl, her dog and the boat are 60.0kg, 30.0kg and 100.0kg respectively. The boat is at rest in the middle of the lake. Calculate the centre of mass of the system. If the dog moves to the other end of the boat,, the girl at the same place, how far and in what direction does the boat move?

Solution:

Let's introduce designations: $M = 100$ kg, $m = 60$ kg and $\mu = 30$ kg, $l = 10$ m.

If zero point of X axis corresponds to the left end of the boat, the center of mass position

$$x_c = \frac{0.5lM}{M + m + \mu} = \frac{500}{100 + 60 + 30} = 2.6 \text{ (m)}.$$

Due to the lack of forces acting in horizontal plane, the position of the center of mass doesn't change, therefore the boat movement δ caused by the dog displacement, corresponds to the

formula: $\dfrac{-m\delta + M(0.5l - \delta) + \mu(l - \delta)}{M + m + \mu} = x_c$ what means $\delta = \dfrac{\mu l}{M + m + \mu} = \dfrac{300}{190} = 1.6$ (m).

The answer:

Center of mass position $x_c = 2.6$ m;

The boat movement $\delta = 1.6$ m, the boat moves in left direction.