**Question #80166, Physics / Other**

The energy of a particle of mass m bound by a special kind of spring is

E=P*2/2m + kx*4

where k is a positive constant. Use the Heisenberg's Uncertainty Principle to calculate the minimum possible energy of the particle.

**Solution**

From uncertainty principle let

$$p\,x \approx \frac{\hbar}{2}$$

$$p = \frac{\hbar}{2x}$$

Then,

$$E = \frac{\left(\frac{\hbar}{2x}\right)^2}{2m} + kx^4 = \frac{\hbar^2}{8mx^2} + kx^4$$

Then we differentiate to find location of minimum E:

$$\frac{dE}{dx} = -\frac{2\hbar^2}{8mx^3} + 4kx^3 = 0$$

So,

$$x = \sqrt[6]{\frac{\hbar^2}{16mk}}$$

Substituting this into the energy equation to find minimum energy:

$$E_0 = \frac{\hbar^2}{8m\sqrt[3]{\frac{\hbar^2}{16mk}}} + k\left(\frac{\hbar^2}{16mk}\right)^{\frac{2}{3}}$$