Answer on Question #79789 - Physics – Mechanics, Relativity

The rectangular components of acceleration for a particle is a_x = 3t and a_y= 30 - 10t, where a is in m/s^2. If the particle start from rest from the origin, find the radius of curvature of the path at t=2 sec.

Solution.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Initial conditions:

$$v_{0.x} = 0;\ v_{0.y} = 0$$

$$v_x(t) = \int a_x\,dt = \int 3t\,dt = \frac{3}{2}t^2 + v_{0.x} = \frac{3}{2}t^2$$

$$v_y(t) = \int a_y\,dt = \int (30 - 10t)dt = 30t - 5\,t^2 + v_{0.y} = 30t - 5t^2$$

$$v^2 = v_x^2 + v_y^2$$

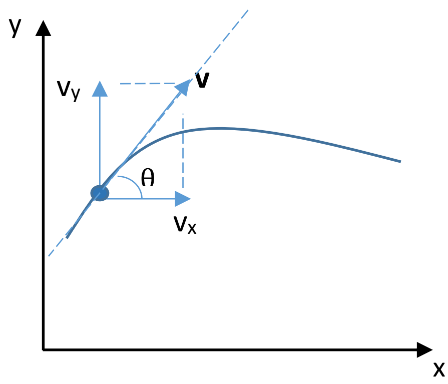
$$v_x(2) = 6\,m/s;\ v_y(2) = 40\,m/s$$

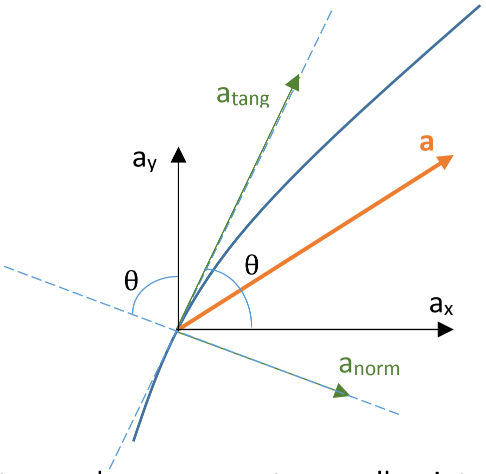
$$v^2(2) = 6^2 + 40^2 = 1636\,m^2/s^2$$

The velocity of the particle is directed along the tangent to its trajectory.

The angle θ between $\vec{v}$ and the x-axis is:

$$\cos\theta = \frac{v_x(2)}{v(2)} = \frac{6}{\sqrt{1636}} = \frac{3}{\sqrt{409}};\ \sin\theta = \frac{v_y(2)}{v(2)} = \frac{40}{\sqrt{1636}} = \frac{20}{\sqrt{409}}$$





The acceleration vector can be decomposed into x and y components as well as into tangential and normal components (parallel and perpendicular to the tangent to the trajectory of motion). The normal component of the acceleration correlates with the radius of curvature as follows:

$$a_{norm} = \frac{v^2}{R}$$

It can be expressed in terms of x and y components of the acceleration:

$$a_{norm} = a_x \times \cos(90° - \theta) - a_y \times \cos\theta$$

$$a_x(2) = 6\ m/s^2;\ a_y(2) = 10\ m/s^2$$

$$a_{norm}(2) = a_x(2) \times \sin\theta - a_y(2) \times \cos\theta = \frac{6 \times 20 - 10 \times 3}{\sqrt{409}} \frac{m}{s^2}$$

$$a_{norm}(2) = \frac{90}{\sqrt{409}}\ m/s^2$$

We obtain:

$$R = \frac{v^2(2)}{a_{norm}(2)} = \frac{1636 \times \sqrt{409}}{90} = \frac{2 \times 409^{3/2}}{45}\ m \approx 367.6\ m$$

Answer: 367.6 m