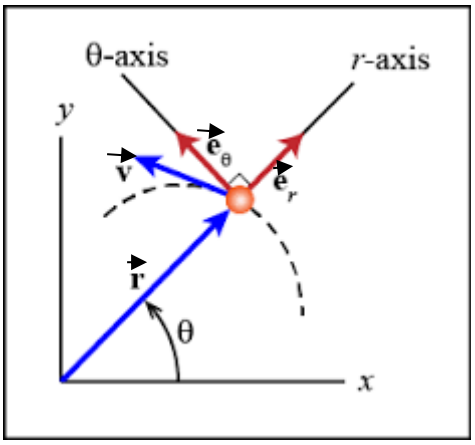
Answer on Question #79788, Physics /Mechanics |Relativity

The motion of the particle is defined by the parametric equations r = 4t^2 and θ = 4t^2 t is in sec, r in meter and ˈθ in radians. After 2 sec, what is the velocity and acceleration vector?

Solution



The position vector of a particle is:

$$\vec{\boldsymbol{r}} = r\overrightarrow{\boldsymbol{e_r}}$$

The velocity vector:

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{dt} = \frac{d(r\overrightarrow{\boldsymbol{e_r}})}{dt} = \frac{dr}{dt}\overrightarrow{\boldsymbol{e_r}} + r\left(\frac{d\overrightarrow{\boldsymbol{e_r}}}{dt}\right)$$

As

$$\frac{d\overrightarrow{\boldsymbol{e_r}}}{dt} = \frac{d\theta}{dt}\overrightarrow{\boldsymbol{e_\theta}}$$

$$\frac{d\overrightarrow{\boldsymbol{e_\theta}}}{dt} = -\frac{d\theta}{dt}\overrightarrow{\boldsymbol{e_r}}$$

then

$$\vec{\boldsymbol{v}} = \frac{dr}{dt}\overrightarrow{\boldsymbol{e_r}} + r\frac{d\theta}{dt}\overrightarrow{\boldsymbol{e_\theta}}$$

$$r = 4t^2, \frac{dr}{dt} = 8t,$$

$$\theta = 4t^2, \frac{d\theta}{dt} = 8t$$

$$\vec{\boldsymbol{v}} = 8t\overrightarrow{\boldsymbol{e_r}} + (4t^2)8t\overrightarrow{\boldsymbol{e_\theta}} = 8t\overrightarrow{\boldsymbol{e_r}} + (32t^3)\overrightarrow{\boldsymbol{e_\theta}}$$

After t=2 sec, $\vec{\boldsymbol{v}} = 8t\overrightarrow{\boldsymbol{e_r}} + (32t^3)\overrightarrow{\boldsymbol{e_\theta}} = 8 \times 2 \times \overrightarrow{\boldsymbol{e_r}} + (32 \times 2^3) \times \overrightarrow{\boldsymbol{e_\theta}} = 16\overrightarrow{\boldsymbol{e_r}} + 256\overrightarrow{\boldsymbol{e_\theta}}$

The acceleration vector:

$$\vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{d\left(\left(\frac{dr}{dt}\right)\overrightarrow{\boldsymbol{e_r}} + r\left(\frac{d\theta}{dt}\right)\overrightarrow{\boldsymbol{e_\theta}}\right)}{dt}$$

$$= \left(\left(\frac{d^2r}{dt^2}\right) - r\left(\frac{d\theta}{dt}\right)^2\right)\overrightarrow{\boldsymbol{e_r}} + \left(r\left(\frac{d^2\theta}{dt^2}\right) + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\right)\overrightarrow{\boldsymbol{e_\theta}}$$

$r = 4t^2, \frac{dr}{dt} = 8t, \frac{d^2r}{dt^2} = 8$

$\theta = 4t^2, \frac{d\theta}{dt} = 8t, \frac{d^2\theta}{dt^2} = 8$

$$\vec{\boldsymbol{a}} = \left(\left(\frac{d^2r}{dt^2}\right) - r\left(\frac{d\theta}{dt}\right)^2\right)\overrightarrow{\boldsymbol{e_r}} + \left(r\left(\frac{d^2\theta}{dt^2}\right) + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\right)\overrightarrow{\boldsymbol{e_\theta}}$$

$$= (8 - 4t^2 \times (8t)^2)\overrightarrow{\boldsymbol{e_r}} + \left(4t^2 \times 8 + 2 \times (8t) \times (8t)\right)\overrightarrow{\boldsymbol{e_\theta}}$$

$$= (8 - 256t^4)\overrightarrow{\boldsymbol{e_r}} + (160t^2)\overrightarrow{\boldsymbol{e_\theta}}$$

After t=2 sec, $\vec{\boldsymbol{a}} = (8 - 256 \times 2^4)\overrightarrow{\boldsymbol{e_r}} + (160 \times 2^2)\overrightarrow{\boldsymbol{e_\theta}} = -4088\overrightarrow{\boldsymbol{e_r}} + 640\overrightarrow{\boldsymbol{e_\theta}}$

Answer: $\vec{\boldsymbol{v}} = 16\overrightarrow{\boldsymbol{e_r}} + 256\overrightarrow{\boldsymbol{e_\theta}}$

$$\vec{\boldsymbol{a}} = -4088\overrightarrow{\boldsymbol{e_r}} + 640\overrightarrow{\boldsymbol{e_\theta}}$$