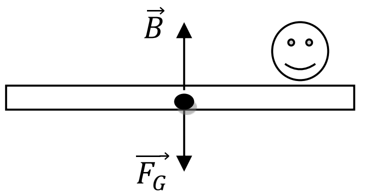
## Answer on Question #79716, Physics/Mechanics|Relativity

A raft of mass M with a man of mass ‘m’ aboard stays motionless on the surface of a lake. The man moves ‘l’ relative to the raft with velocity V(t) and then stops. Assuming the water resistance be negligible, find

1. Displacement of the raft relative to the shore.
2. Horizontal component of the force with which the man acted on the raft during the motion.

**Solution**

1. Consider system of the “raft-man” as one, then the acting external forces are:



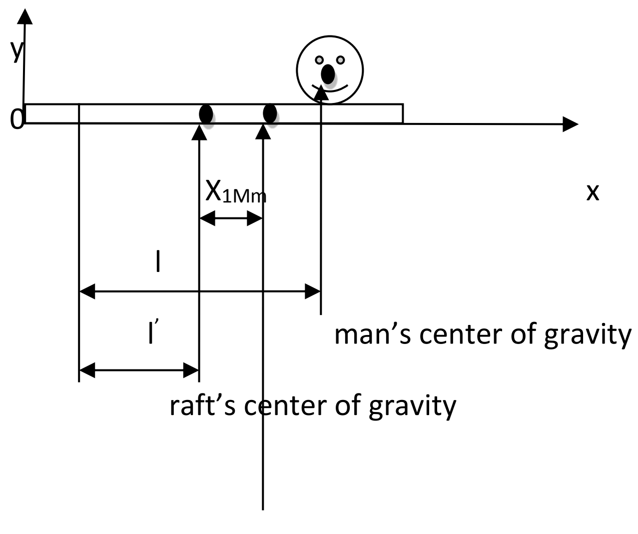

$\vec{F_G}$ force of gravity pointing vertically up

$\vec{B}$ buoyancy force pointing vertically up

Cause of the raft’s movement in the horizontal way is a friction force between a man’s shoes and a surface of raft.

As the resistance of water is negligible, the resultant of all external forces acting on the system “raft-man” is equal to zero. The position of the center of mass of a system “raft-man” does not change in the process of motion.

Find the center of gravity at rest at the beginning



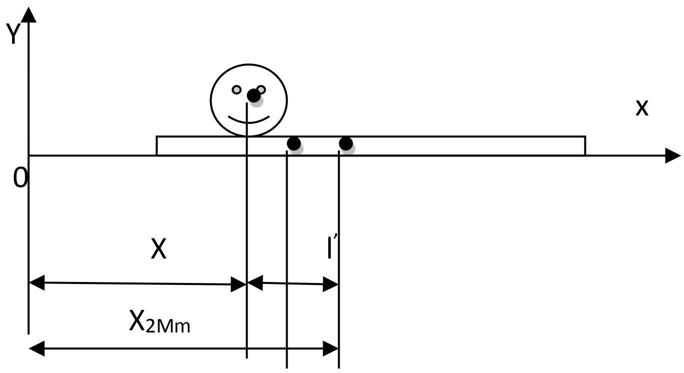

center of mass of the system “raft-man”

For the x –coordinate of the center of gravity in the system of n particles is formula:

$$X_{1Mm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

$$X_{1Mm} = \frac{Ml' + ml}{M + m}$$

After a man's walk




$$X_{2Mm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{mX + M(X + l')}{M + m}$$

System's position of the center of gravity remains the same during the walk, thus

$$X_{1Mm} = X_{2Mm}$$

$$\frac{Ml' + ml}{M + m} = \frac{mX + M(X + l')}{M + m}$$

$$Ml' + ml = mX + M(X + l')$$

$$Ml' + ml = mX + MX + Ml'$$

$$ml = mX + MX$$

$$X = \frac{ml}{m + M}$$

2. As net external force on "raft-man" system is equal to zero, therefore the momentum of this system does not change.

$$0 = m\left[\vec{v}(t) + \overrightarrow{v'}(t)\right] + M\overrightarrow{v'}(t)$$

$$0 = m\,\vec{v}(t) + m\overrightarrow{v'}(t) + M\overrightarrow{v'}(t)$$

$$\overrightarrow{v'}(t) = -\frac{m\vec{v}(t)}{M + m}$$

As $\vec{v}(t), \overrightarrow{v'}(t)$ are along x axis, but $v(t)$ has opposite direction, then

$$v'(t) = \frac{mv(t)}{M + m}$$

$$F_x = M\frac{dv'(t)}{dt} = \frac{Mm}{M + m} \times \frac{dv(t)}{dt}$$

**Answer:** **1.** $\boldsymbol{X = \frac{ml}{m+M}}$,

**2.** $\boldsymbol{F_x = \frac{Mm}{M+m} \times \frac{dv(t)}{dt}}$