If r is the horizontal range of a projectile; and h is its greatest height, prove that initial speed is $[2g(h+(r^2/16h))]^{1/2}$.

If maximum height is h, the time to reach this height (and the time to fall than down) is

$$t = \sqrt{\frac{2h}{g}}$$

The range is

$$r=2v_h\sqrt{\frac{2h}{g}}$$

The square of the horizontal speed is

$$v_h^2 = \frac{r^2 g}{8 \cdot h}$$

and the square of the vertical speed is

$$v_v^2 = 2 \cdot g \cdot h$$

Finally, the initial speed is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{\frac{r^2g}{8 \cdot h} + 2 \cdot g \cdot h} = \sqrt{2g\left(\frac{r^2g}{8 \cdot h} + h\right)}$$

Answer provided by https://www.AssignmentExpert.com