Answer on Question #79618, Physics / Mechanics | Relativity

Evaluate the output motion in a slider-crank mechanism with uniform input motion of the crank, for compliance with the conditions necessary for it to describe simple harmonic motion

Solution

Agree that crank's angular velocity is ω. Then crank angle is

$$\alpha = \omega \cdot t$$

Where t – time which the mechanism works.

Consider triangle made by crank radius of length R, rod of length L and X – position of piston pin from crank centre.

According to cosine theorem:

$$L^2 = X^2 + R^2 - 2 \cdot X \cdot R \cdot \cos(wt),$$

$$X = R \cdot \cos(wt) + \sqrt{L^2 - (R \cdot \sin(wt))^2},$$

The output velocity:

$$v = \frac{dX}{dt} = -R \cdot \omega \cdot \sin(\omega t) - \frac{R^2 \cdot \omega \cdot \sin(\omega t) \cdot \cos(\omega t)}{\sqrt{L^2 - (R \cdot \sin(wt))^2}}$$

The output acceleration:

$$a = \frac{dv}{dt} = -R \cdot \omega^2 \cdot \cos(\omega t) - \frac{R^2 \cdot \omega^2 \cdot \cos^2(\omega t)}{\sqrt{L^2 - (R \cdot \sin(wt))^2}} - \frac{R^4 \cdot \omega^2 \cdot \cos^2(\omega t) \cdot \sin^2(\omega t)}{\sqrt{(L^2 - (R \cdot \sin(wt))^2)^3}}.$$

The output motion is not harmonic. In case $L = \infty$ the output motion will be harmonic:

$$X = R \cdot \cos(\omega t)\,, v = -R\omega \cdot \sin(\omega t)\,, a = -R\omega^2 \cdot \cos(\omega t).$$

Answer

$$X = R \cdot \cos(wt) + \sqrt{L^2 - (R \cdot \sin(wt))^2}$$

$$v = \frac{dX}{dt} = -R \cdot \omega \cdot \sin(\omega t) - \frac{R^2 \cdot \omega \cdot \sin(\omega t) \cdot \cos(\omega t)}{\sqrt{L^2 - (R \cdot \sin(wt))^2}}$$

$$a = \frac{dv}{dt} = -R \cdot \omega^2 \cdot \cos(\omega t) - \frac{R^2 \cdot \omega^2 \cdot \cos^2(\omega t)}{\sqrt{L^2 - (R \cdot \sin(wt))^2}} - \frac{R^4 \cdot \omega^2 \cdot \cos^2(\omega t) \cdot \sin^2(\omega t)}{\sqrt{(L^2 - (R \cdot \sin(wt))^2)^3}}.$$