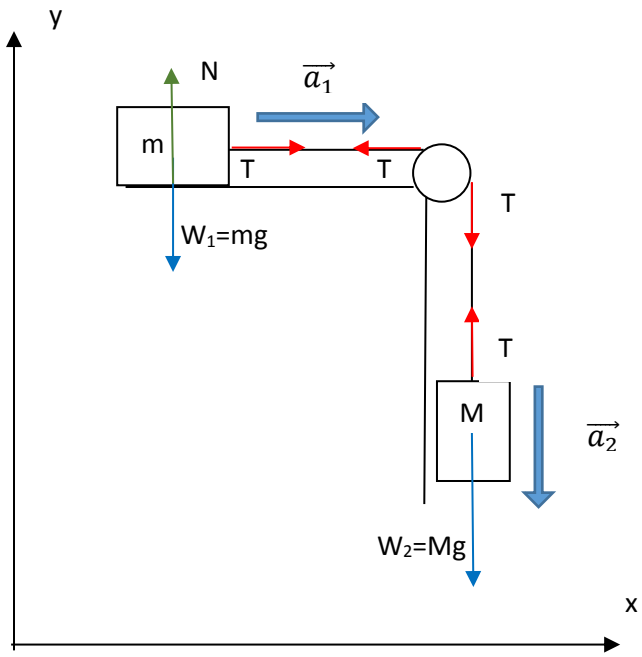
Answer on Question #79522 - Physics - Mechanics, Relativity

We placed an object of mass m [kg] on a horizontal table, attached a thread to it and hung a weight M [kg] at the other end of the thread through a light constant pulley. Find the acceleration a [m / s^2] of the object and the weight and the tension T [N] of the thread. However, it is assumed that the friction of the pulley and the mass of the pulley and the yarn are negligible, and it is assumed to be the gravitational acceleration g [m / s^2].

Solution.



Let us direct the axes of the Cartesian coordinate system as shown in the figure.

Write the 2nd Newton's law for each of the two bodies:

$$\overrightarrow{F_1} = m\overrightarrow{a_1};\ \ \overrightarrow{F_2} = M\overrightarrow{a_2},$$

where $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are the vector sums of all the forces acting on the objects with masses m and M respectively, $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ are the accelerations of these objects.

On the object of mass m three forces act: the force of gravity (or weight) $\overrightarrow{W_1} = m\vec{g}$, the normal force $\vec{N}$ and the tension $\vec{T}$. $\overrightarrow{W_1}$ and $\vec{N}$ are directed along the y axis, $\vec{T}$ is directed along the x axis (see figure). Since the object moves along a horizontal surface, the acceleration $\overrightarrow{a_1}$ has only the x component.

On the weight M two forces act: the force of gravity $\overrightarrow{W_2} = M\vec{g}$ and the tension $\vec{T}$. The tension value is the same along the entire thread (we assume that it is inextensible).

$\overrightarrow{W_2}$, $\vec{T}$ and $\overrightarrow{a_2}$ are directed along the y-axis as shown in the figure.

The 2nd law of Newton for two bodies can be written as

$$\begin{cases} \overrightarrow{W_1} + \vec{N} + \vec{T} = m\vec{g} + \vec{N} + \vec{T} = m\overrightarrow{a_1} \\ \overrightarrow{W_2} + \vec{T} = M\vec{g} + \vec{T} = M\overrightarrow{a_2} \end{cases}$$

Since friction of the pulley is negligible, the acceleration of the objects with masses m and M are equal in absolute value: $|\overrightarrow{a_1}| = |\overrightarrow{a_2}|$.

Let us write the component form of Newton's second law. It is sufficient to write down the expressions only for the x-component for the body of mass m and for the y-component for the weight M

$$\begin{cases} F_{1x} = ma_1 = ma \\ F_{2y} = Ma_2 = Ma \end{cases}$$

$$F_{1x} = T; \;\; F_{2y} = -Mg + T$$

We receive a system of equations:

$$\begin{cases} T = ma \\ T - Mg = Ma \end{cases}$$

After substituting T from the first equation to the second, we get:

$$ma - Mg = Ma$$

$$a = \frac{Mg}{m - M} \; [m/s^2]$$

Then we can obtain T:

$$T = ma = \frac{mMg}{m - M} \; [N]$$

Answer: $a = \frac{Mg}{m-M} \; [m/s^2]; T = \frac{mMg}{m-M} \; [N]$