Answer on Question #79255 – Physics/Mechanics - Relativity

A 60 kg man running at an initial speed of 4 m/s jumps onto a 120 kg cart initially at rest. He slides on the cart's top surface and finally comes to rest relative to the cart. The coefficient of kinetic friction between the man and the cart is 0.4. Friction between the cart and the ground can be neglected.

I) Find the final speed of the man and cart relative to the ground.

II) Find the frictional force acting on the man while he is sliding across the top surface of the cart.

III) How long does the frictional force act on him?

IV) Find the change in momentum of the man and the change in momentum of the cart.

V) Determine the displacement of the man relative to the ground while he is sliding on the cart.

Solution

Let the man's mass and initial speed be m and v, cart's mass and final speed M and u.

I) When the man reaches the cart, he delivers it part of his momentum and they move together:

$$mv = (M + m)u,$$

$$u = \frac{mv}{M + m} = \frac{60 \cdot 4}{120 + 60} = 1.33 \text{ m/s}.$$

II) The frictional force acting on the man:

$$f_{\text{k}} = \mu \cdot N = \mu \cdot mg = 0.4 \cdot 60 \cdot 9.8 = 235.2 \text{ N}.$$

III) According to the law of conservation of energy man's initial kinetic energy is spent on motion of the man and the cart and work done by friction on distance x:

$$\frac{1}{2}mv^2 = \frac{1}{2}(M + m)u^2 + \mu mgx,$$

$$x = \frac{Mv^2}{2\mu g(M + m)}.$$

Since after reaching the cart man's deceleration was uniform, the frictional force action time is:

$$t = \frac{2x}{v} = \frac{Mv}{\mu g(M + m)} = \frac{120 \cdot 4}{0.4 \cdot 9.8 \cdot (120 + 60)} = 0.68 \text{ s}.$$

IV) Change in momentum of the man is difference between his final momentum and initial momentum:

$$\Delta p_m = p_{m2} - p_{m1} = mu - mv = 60(1.33 - 4) = -160.2 \text{ kg·m/s}.$$

Change in momentum of the cart:

$$\Delta p_c = p_{c2} - p_{c1} = Mu - 0 = 120 \cdot 1.33 - 0 = 159.6 \text{ kg·m/s}.$$

V) Meanwhile the man decelerated from v to 0 due to friction, the cart was accelerating from 0 to u, thus the displacement is

$$d = x + s = \frac{t}{2}(v + u) = \frac{0.68}{2}(4 + 1.33) = 1.81 \text{ m}.$$

Answer: I) 1.33 m/s; II) 235.2 N; III) 0.68 s; IV) -160.2 kg·m/s, 159.6 kg·m/s; V) 1.81 m