Answer on Question#78184 - Physics - Mechanics - Relativity

A particle moves under the action of a central force. Show that:

(i) its angular momentum is constant and

(ii) its motion is confined to a plane.

Solution:

The equation of circular motion of a body under a torque $\vec{\tau}$ is given by

$$\vec{\tau} = \frac{d\vec{L}}{dt},$$

where $\vec{L}$ – angular momentum of the body.

Since a central force $\overrightarrow{F_c}$ is parallel to the radius-vector $\vec{r}$ of the body (directed from the force's center to the body), its torque is zero:

$$\overrightarrow{\tau_c} = \vec{r} \times \overrightarrow{F_c} = \vec{0}$$

Thus

$$\frac{d\vec{L}}{dt} = \overrightarrow{\tau_c} = \vec{0} \;\Rightarrow\; \vec{L} = const$$

Since $\vec{L}$ is constant, it doesn't change it direction. It means that the motion of the body is confined to a plane perpendicular to vector $\vec{L}$, otherwise the direction of $\vec{L}$ will change, which is forbidden by the mentioned condition.