Answer on Question#78165 - Physics - Mechanics - Relativity

The vibrations of a string fixed at both ends are represented by the equation $y(x,t) = 2\sin(\pi x/3)\cos(50\pi t)$ meter. This stationary wave is produced due to superposition of $y_1(x,t) = A\sin\frac{2\pi}{\lambda}(x - vt)$ and $y_2(x,t) = A\sin\frac{2\pi}{\lambda}(x + vt)$.

(i) Obtain the equations of component waves, and

(ii) calculate the distance between two consecutive nodes of the stationary wave.

Solution:

It's known that

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

Thus

$$y_1(x,t) + y_2(x,t) = 2A\sin\frac{2\pi}{\lambda}x\cos\frac{2\pi}{\lambda}vt$$

Since $y(x,t) = y_1(x,t) + y_2(x,t)$, we obtain

$$2\sin(\pi x/3)\cos(50\pi t) = 2A\sin\frac{2\pi}{\lambda}x\cos\frac{2\pi}{\lambda}vt$$

Therefore

$$A = 1\text{ m},\quad \lambda = 6\text{ m},\quad v = 150\,\frac{\text{m}}{\text{s}}$$

The equations of components:

$$y_1(x,t) = \sin\frac{\pi}{3}(x - 150t)$$

$$y_2(x,t) = \sin\frac{\pi}{3}(x + 150t)$$

The distance between two consecutive nodes d is given by the half of the wavelength λ:

$$d = \frac{\lambda}{2} = 3\text{ m}$$

Answer:

(i) $y_1(x,t) = \sin\frac{\pi}{3}(x - 150t)$,

$y_2(x,t) = \sin\frac{\pi}{3}(x + 150t)$

(ii) 3 m.