Answer on Question #78161, Physics / Mechanics | Relativity

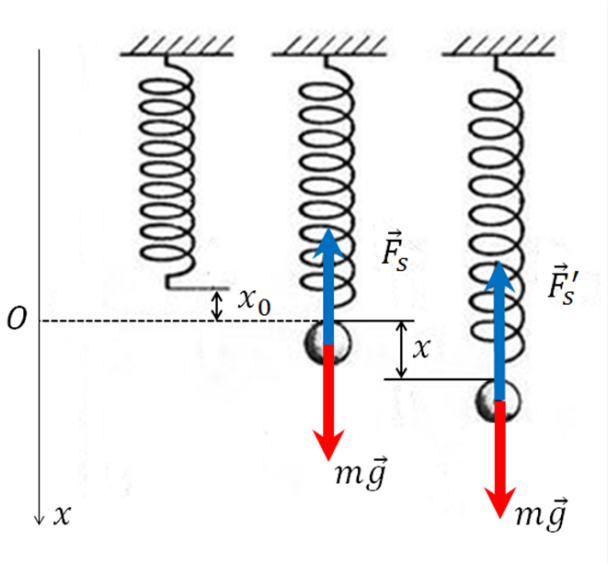
Question. A body of mass $0.2\ kg$ is suspended from a spring of force constant $100\ N/m$. The frictional force due to air drag acting on it is $5v$ newton. (1) Write down the differential equation of motion and calculate the period of free oscillations. (2) If a harmonic force $F = 20\cos 20t$ is applied on it, calculate the phase lag in the steady state.

Solution

(1) So,

According to the Second Newton's Low

$$F = ma$$



We have

$$mg = kx_0$$

$$F_s' = -k(x_0 + x)$$

$$ma_x = -k(x_0 + x) + mg - 5v = -kx_0 - kx + kx_0 - 5v = -kx - 5v$$

$$m\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{5}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\boldsymbol{\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0},$$

where $\beta = \frac{5}{2m} = \frac{5}{2 \cdot 0.2} = 12.5 \ \frac{1}{s}$ and $\omega_0^2 = \frac{k}{m} = \frac{100}{0.2} = 500 \ \frac{1}{s^2}$

$$\boldsymbol{\frac{d^2x}{dt^2} + 25\frac{dx}{dt} + 500x = 0},$$

For this system

$$\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{500 - (12.5)^2} = 18.54 \ \frac{1}{s}$$

$$T = \frac{2\pi}{\omega} = \frac{2 \cdot 3.14}{18.54} = 0.34 \ s$$

Answer. $\frac{d^2x}{dt^2} + 25\frac{dx}{dt} + 500x = 0, T = 0.34 \ s.$

(2)

Using (1) we get

$$m\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + kx = 20\cos 20t \ \rightarrow \frac{m}{k}\frac{d^2x}{dt^2} + \frac{5}{k}\frac{dx}{dt} + x = 20/k\cos 20t \ \rightarrow$$

$$\omega_0 = \sqrt{\frac{k}{m}}, \ \gamma = \frac{5}{2\sqrt{km}}, \ \ K = \frac{1}{k}$$

$$\frac{1}{\omega_0^2}\frac{d^2x}{dt^2} + \frac{2\gamma}{\omega_0}\frac{dx}{dt} + x = K \cdot 20\cos 20t$$

Steady State solution

$$x(t) = X_0 \sin(\omega t + \varphi)$$

$$X_0 = \frac{KF_0}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\gamma\omega}{\omega_0}\right)^2}}$$

$$\varphi = \tan^{-1}\frac{-2\gamma\omega/\omega_0}{1 - \frac{\omega^2}{\omega_0^2}} = \tan^{-1}\frac{-2 \cdot 0.56 \cdot 20/22.36}{1 - \frac{20^2}{22.36^2}} = -78.72°$$

Answer. $\varphi = -78.72°.$