Answer on Question#78151 - Physics - Mechanics - Relativity

Solid sphere of mass M and radius R starts from rest and rolls without slipping down an inclined plane from a point A. Determine the velocity with which it will reach a point B at a vertical distance h from point A.

Solution:

According to the law of conservation of energy the difference in potential energy between points A and B must be equal to the sum of gained translational kinetic energy ($E_t = \frac{1}{2}I\omega^2$) and angular kinetic energy ($E_a = \frac{1}{2}Mv^2$):

$$Mgh = E_t + E_a = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2,$$

where g – acceleration due to gravity, I – moment of inertia of the sphere, ω – angular velocity of the sphere, v – velocity of the sphere.

Since the sphere is hollow its moment of inertia is given by

$$I = \frac{2}{3}MR^2$$

Also it is given that the sphere rolls without slipping, thus

$$\omega = \frac{v}{R}$$

Therefore

$$Mgh = \frac{1}{2}\cdot\frac{2}{3}MR^2\left(\frac{v}{R}\right)^2 + \frac{1}{2}Mv^2$$

$$v = \sqrt{\frac{6}{5}gh}$$

Answer: $v = \sqrt{\frac{6}{5}gh}$.