Question 78001

A particle travels a distance s in time t. It starts from rest and in the first part of the journey it moves with constant acceleration a and in the second part with a constant retardation b till it comes to rest. Show that ab(t^2) = 2s(a+b)

Solution

Let's consider 2 periods of time - t_1, when body is accelerating and t_2, when body is retarding.

We can define velocity of body at the end of the journey, as $v = at_1 - bt_2$. But it is mentioned, that body is resting at the end, so $v = 0$. We can define position of body at the end of the journey, as:

$S = S_1 + S_2 = (\frac{at_1{}^2}{2}) + (v_{12}t_2 - \frac{bt_2^2}{2})$, where v_{12} – is a velocity of body, at the end of acceleration period. So $v_{12} = at_1$, and $S = \frac{at_1{}^2}{2} + at_1t_2 - \frac{bt_2^2}{2}$. And of course, $t_1 + t_2 = t$.

$$\begin{cases} at_1 - bt_2 = 0 \\ S = \dfrac{at_1{}^2}{2} + at_1t_2 - \dfrac{bt_2^2}{2} \\ t_1 + t_2 = t \end{cases}; \begin{cases} t_1 = t - t_2 \\ a(t - t_2) = bt_2 \\ \dfrac{a(t-t_2)^2}{2} + at_2(t - t_2) - \dfrac{bt_2^2}{2} = S \end{cases}; \begin{cases} at = t_2(a+b) \\ \dfrac{at^2}{2} - \dfrac{2att_2}{2} + \dfrac{at_2^2}{2} + at_2t - at_2^2 - \dfrac{bt_2^2}{2} = S \end{cases}$$

$$\begin{cases} t_2 = \dfrac{at}{a+b} \\ \dfrac{at^2}{2} - \dfrac{at_2^2}{2} - \dfrac{bt_2^2}{2} = S \end{cases}; 2S = at^2 - t_2^2(a+b) = at^2 - \frac{a^2t^2}{(a+b)^2}(a+b)$$

$$2S(a+b) = at^2(a+b) - a^2t^2 = abt^2$$

$$2S(a+b) = abt^2$$

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