Answer on Question #76698, Physics / Other

A block of mass 1 kg is attached to a spring of force constant k = 25/4 N/m. It is pulled 0.3 m from its equilibrium position and released from rest. This spring-block apparatus is submerged in a viscous fluid medium which exerts a damping force of -4 v (where v is the instantaneous velocity of the block). Determine of the position x(t) of the block at time t.

Solution:

According to Newton's second law,

$$ma = -kx + F_{frict}$$

$$a = \ddot{x}, \qquad F_{frict} = -4v = -4\dot{x}$$

Thus,

$$m\ddot{x} + 4\dot{x} + kx = 0$$

We obtain the following differential equation:

$$\ddot{x} + 4\dot{x} + \frac{25}{4}x = 0$$

Initial conditions:

$$\begin{cases} x(0) = 0.3\ m \\ \dot{x}(0) = 0 \end{cases}$$

Assume a solution of the form

$$x = e^{\lambda t}$$

we get

$$\lambda^2 + 4\lambda + \frac{25}{4} = 0$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 - 25}}{2} = -2 \pm 1.5i$$

so the displacement is:

$$x(t) = C_1 e^{-2t}\cos(1.5t) + C_2 e^{-2t}\sin(1.5t)$$

$$\dot{x}(t) = C_1 e^{-2t}(-2\cos(1.5t) - 1.5\sin(1.5t)) + C_2 e^{-2t}(-2\sin(1.5t) + 1.5\cos(1.5t))$$

We use initial conditions to find C_1 and C_2:

$$C_1\cos(0) + C_2\sin(0) = 0.3$$

$$C_1(-2\cos(0) - 1.5\sin(0)) + C_2(-2\sin(0) + 1.5\cos(0)) = 0$$

$$C_1 \cdot 1 + C_2 \cdot 0 = 0.3$$

$$C_1(-2) + C_2(1.5) = 0$$

So,

$$C_1 = 0.3$$

$$C_2 = \frac{2 \cdot 0.3}{1.5} = 0.4$$

Then the position x(t) of the block at time t

$$x(t) = 0.3e^{-2t}\cos(1.5t) + 0.4e^{-2t}\sin(1.5t)$$

Answer: $x(t) = 0.3e^{-2t}\cos(1.5t) + 0.4e^{-2t}\sin(1.5t)$.