# Answer on Question # 76071, Physics -Mechanics- Relativity:

**Question:** The equation of motion of a damped harmonic oscillator is given by

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$$

with m = 0.25kg, b = 0.14 $sec^{-1}$ and $\omega_0$ =18.4 $sec^{-1}$.

Calculate i) the time period; ii) number of oscillations in which its amplitude will become half of its initial value; and iii) number of oscillations in which its mechanical energy will reduce to half of its initial value.

**Solution:** The equation of motion of a damped harmonic oscillator is given by

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0 \quad \text{..........................................(1)}$$

Given ,mass (m)=0.25 kg

Damping constant (b)= 0.14 $sec^{-1}$

Damping frequency ($\omega_0$) =18.4 $sec^{-1}$

(i). solution of the equation (1) is x(t)= A exp(-bt).Cos $\omega_0$t .........................(2)

Or, x (t) = A exp(-0.14t).Cos $\omega_0$t

Now amplitude = A exp(-0.14t)

So, time period = $\frac{2\pi}{\omega_0} = \frac{2\pi}{18.4} = 0.341$ sec.

(ii). Amplitude becomes half of its initial value i.e. exp(-0.14t) = $\frac{1}{2}$ or, t= 4.95 sec.

So, the number of oscillation = $\frac{4.95}{0.341} = 14.52$

(iii). Mechanical energy of the oscillator is $\frac{1}{2}$ x $\omega_0$ x(amplitude)$^2$ = $\frac{1}{2}$ x 18.4 x(A)$^2$ x exp(-0.28t)

Now mechanical energy becomes half of its initial value i.e. exp(-0.28t) = $\frac{1}{2}$ or, t =2.47 sec.

So, the number of oscillation = $\frac{2.47}{0.341} = 7.24$

**Answer:** (i). 0.341 sec.

(ii). 14.52

(iii). 7.24