Answer on Question #75888, Physics / Mechanics | Relativity

Find the elevation h (km) where the weight of an object one-tenth its weight on the surface of the earth.

Solution

To answer this question we should use the Law of Universal Gravitation:

$$F_{GRAV} = \frac{Gm_1m_2}{r^2}$$

We have two objects: the Earth and an object, as the Earth has significant size r – is distance between centers of these objects. Radius of Earth is R_E = 6378 km. The unknown elevation is h. Then $r = R_E + h$ and formula for F_{GRAV} is:

$$F_{GRAV} = \frac{Gm_Em_2}{(R_E + h)^2}$$

Gravitation force acting on object is it's weight $w = F_{GRAV}$
Weight of an object the surface of the earth is:

$$w = \frac{Gm_Em_2}{R_E^{\,2}}$$

Weight of an object at elevation h is:

$$w_h = \frac{Gm_Em_2}{(R_E + h)^2}$$

We know that $w_h = w/10$, then:

$$\frac{Gm_Em_2}{(R_E + h)^2} = \frac{Gm_Em_2}{10\cdot R_E^2};$$

$$\frac{10\cdot R_E^2}{(R_E + h)^2} = 1;$$

$$\left(\frac{\sqrt{10}\cdot R_E}{(R_E+h)}\right)^2 - 1 = 0$$

$$\frac{\sqrt{10}\cdot R_E}{R_E+h} - 1 = 0 \quad \text{or} \quad \frac{\sqrt{10}\cdot R_E}{R_E+h} + 1 = 0;$$

$\sqrt{10}\cdot R_E - R_E - h = 0$ or $\sqrt{10}\cdot R_E + R_E + h = 0$;

$2.16R_E = h$ or $4.16R_E = -h$;

As $h > 0$, $h = 2.16R_E = 2.16 \cdot 6378 = 13776.48$ (km).

Answer: 13776.48 km