Answer on Question #75886 Physics / Mechanics | Relativity

A harmonic wave on a rope is described by $y(x,t) = 4.00 \sin\left(\frac{2\pi[10t + x]}{0.82}\right)$

i) Calculate the wavelength and time period of the wave.

ii) Determine the displacement and acceleration of the element of the rope located at $x = 0.58$ m at time, $t = 41.0$ s.

Solution:

i) The general form of a harmonic wave

$$y(x,t) = y_m \sin(\omega t + kx)$$

where ω is an angular frequency, k is a wave number.

So

$$\omega = \frac{2\pi \times 10}{0.82} = 24.4\pi$$

$$k = \frac{2\pi}{0.82}$$

The wavelength

$$\lambda = \frac{2\pi}{k} = 0.82 \text{ m}$$

The period

$$T = \frac{2\pi}{\omega} = 0.082 \text{ s} = 82 \text{ ms}$$

ii) The displacement

$$y(0.58, 41.0) = 4.00 \sin\left(\frac{2\pi[10 \times 41.0 + 0.58]}{0.82}\right) = -3.86 \text{ mm}$$

The acceleration

$$a = y'' = -\omega^2 y$$

So

$$a = -(24.4\pi)^2 \times (-3.86) = 22681 \frac{\text{mm}}{\text{s}^2} = 22.7 \frac{\text{m}}{\text{s}^2}$$

Answers:

i)

0.82 m

82 ms

ii)

-3.86 mm

$22.7\ \frac{\mathrm{m}}{\mathrm{s}^2}$