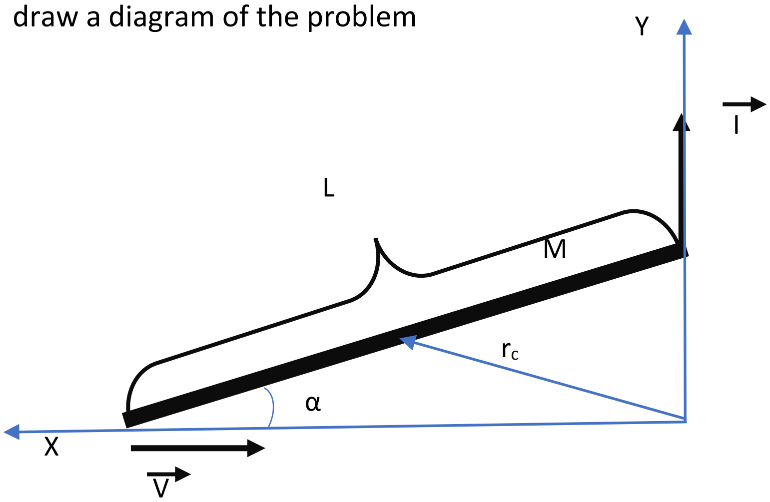
Question #75613

Description:

A uniform rod of mass M and length L is lying on a horizontal frictionless surface if a horizontal impulse I perpendicular to length of the road is applied at one end of the road the other end begins to move with speed V then find the magnitude of the impulse

Solution.

draw a diagram of the problem



r_c- radius vector of the center of mass, which is in the middle of the length of the rod, we design on the coordinate axis

$$r_{cx} = \frac{L}{2}\cos\alpha, \quad r_{cy} = \frac{L}{2}\sin\alpha$$

find the velocity components as derived from the radius of the vector

$$\frac{dr_{cx}}{dt} = v_{cx} = -\frac{L}{2}\sin\alpha\frac{d\alpha}{dt}, \quad \frac{dr_{cy}}{dt} = v_{cy} = \frac{L}{2}\cos\alpha\frac{d\alpha}{dt}$$

horizontally, the lower end moves at a speed v=const, we get (1)

$$L\cos\alpha = L - vt$$

find the derivative (2)

$$\frac{d(L\cos\alpha)}{dx} = -L\sin\alpha\frac{d\alpha}{dt} = -v = 2v_{cx}\,(2), => \quad v_{cx} = -\frac{v}{2}, => \sin\alpha = \frac{v}{L\frac{d\alpha}{dt}}$$

it is obvious that the vertical speed as an opposite to the horizontal speed

$$v_{cy} = -v_{cx}\,\text{cotan}\alpha$$

from equation (1) we find the cotangent

$$\text{cotan}\alpha = \sqrt{\frac{\cos^2\alpha}{1-\cos^2\alpha}} = \sqrt{\frac{(\frac{L-vt}{L})^2}{1-(\frac{L-vt}{L})^2}} = \frac{L-vt}{\sqrt{L^2-L^2+2Lvt-v^2t^2}} = \frac{L-vt}{\sqrt{2Lvt-v^2t^2}}$$

$$v_{cy} = v_{cx}\,\text{cotan}\alpha = \frac{v}{2}\frac{L-vt}{\sqrt{2Lvt-v^2t^2}}$$

then the momentum of the rod is a vector with coordinates

$$\overrightarrow{Mv_c} =< v_{cx}, v_{cy} >=< -M\frac{v}{2}, \; M\frac{v}{2}\frac{L-vt}{\sqrt{2Lvt-v^2t^2}} >$$

$$|\overrightarrow{Mv_c}| = M \; \sqrt{v^2{}_{cx} + v^2{}_{cy}} = ML\frac{v}{2}\,\frac{1}{\sqrt{2Lvt-v^2t^2}}$$

Answer

$$\overrightarrow{Mv_c} =< v_{cx}, v_{cy} >=< -M\frac{v}{2}, \; M\frac{v}{2}\frac{L-vt}{\sqrt{2Lvt-v^2t^2}} >$$

$$|\overrightarrow{Mv_c}| = \sqrt{v^2{}_{cx} + v^2{}_{cy}} = ML\frac{v}{2}\,\frac{1}{\sqrt{2Lvt-v^2t^2}}$$