

Answer on Question 75405, Physics, Mechanics, Relativity

Question:

A child of mass 50 kg is standing on the edge of a merry-go-round of mass 250 kg and radius 3.0 m which is rotating with an angular velocity of 3.0 rad/s . The child then starts walking towards the centre of the merry-go-round. What will be the final angular velocity of the merry-go-round when the child reaches the centre?

Solution:

We can find the final angular velocity of the merry-go-round from the law of conservation of angular momentum:

$$L_i = L_f,$$

$$I_i \omega_i = I_f \omega_f,$$

here, I_i is the initial rotational inertia of the system, I_f is the final rotational inertia of the system, ω_i is the initial angular velocity of the merry-go-round, ω_f is the final angular velocity of the merry-go-round.

We can find the initial rotational inertia of the system as follows:

$$I_i = (I_{disk,i} + I_{child,i}) = \left(\frac{1}{2} m_{disk} r_{disk,i}^2 + m_{child} r_{child,i}^2 \right),$$

here, $I_{disk,i} = \frac{1}{2} m_{disk} r_{disk,i}^2$ is the initial rotational inertia of the merry-go-round, $I_{child,i} = m_{child} r_{child,i}^2$ is the initial rotational inertia of the child, m_{disk} is the mass of the merry-go-round, m_{child} is the mass of the child, r_{disk} is the radius of the merry-go-round, r_{child} is the distance from the centre of the merry-go-round to the child.

Then, we can calculate I_i :

$$\begin{aligned} I_i &= \left(\frac{1}{2} m_{disk} r_{disk,i}^2 + m_{child} r_{child,i}^2 \right) = \\ &= \left(\frac{1}{2} \cdot 250 \text{ kg} \cdot (3.0 \text{ m})^2 + 50 \text{ kg} \cdot (3.0 \text{ m})^2 \right) = 1575 \text{ kg} \cdot \text{m}^2. \end{aligned}$$

Similarly, we can find the final rotational inertia of the system:

$$I_f = (I_{disk,f} + I_{child,f}) = \left(\frac{1}{2} m_{disk} r_{disk,f}^2 + m_{child} r_{child,f}^2 \right),$$

here, $I_{disk,f} = \frac{1}{2}m_{disk}r_{disk,f}^2$ is the final rotational inertia of the merry-go-round, $I_{child,f} = m_{child}r_{child,f}^2$ is the final rotational inertia of the child, m_{disk} is the mass of the merry-go-round, m_{child} is the mass of the child, r_{disk} is the radius of the merry-go-round, r_{child} is the distance from the centre of the merry-go-round to the child.

Then, we can calculate I_f :

$$I_f = \left(\frac{1}{2}m_{disk}r_{disk,f}^2 + m_{child}r_{child,f}^2 \right) = \\ = \left(\frac{1}{2} \cdot 250 \text{ kg} \cdot (3.0 \text{ m})^2 + 50 \text{ kg} \cdot (0.0 \text{ m})^2 \right) = 1125 \text{ kg} \cdot \text{m}^2.$$

Finally, we can calculate the final angular velocity of the merry-go-round from the law of conservation of angular momentum:

$$I_i\omega_i = I_f\omega_f, \\ \omega_f = \omega_i \frac{I_i}{I_f} = 3.0 \frac{\text{rad}}{\text{s}} \cdot \frac{1575 \text{ kg} \cdot \text{m}^2}{1125 \text{ kg} \cdot \text{m}^2} = 4.2 \frac{\text{rad}}{\text{s}}.$$

Answer:

$$\omega_f = 4.2 \frac{\text{rad}}{\text{s}}.$$

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