Answer on Question #75226 Physics / Other

For a particle undergoing circular motion with an angular velocity w in a circle of radius r show that: $\mathbf{w} \times (\mathbf{w} \times \mathbf{r}) = -\mathbf{r}\, w^2$.

Solution:

Using the vector triple product expansion

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

we get

$$\mathbf{w} \times (\mathbf{w} \times \mathbf{r}) = \mathbf{w}(\mathbf{w} \cdot \mathbf{r}) - \mathbf{r}(\mathbf{w} \cdot \mathbf{w})$$

Because

$$\mathbf{w} \perp \mathbf{r}, \qquad \mathbf{w} \parallel \mathbf{w}$$

So

$$(\mathbf{w} \cdot \mathbf{r}) = 0$$

$$(\mathbf{w} \cdot \mathbf{w}) = w^2$$

Finally

$$\mathbf{w} \times (\mathbf{w} \times \mathbf{r}) = -\mathbf{r}\, w^2$$