Answer on Question #75219, Physics / Other

Two waves, travelling along the same direction, are given by

y1(x, t) = asin (w1t − k1x)

and y2 (x, t) = asin (w2t − k2x)

Suppose that the values of w1 and k1 are respectively slightly greater than w2 and k2 .

i) Obtain an expression for the resultant wave due to their superposition.

ii) Explain the formation of wave packet.

Solution:

(i) Let the waves are

$$y_1(x,t) = a\sin(\omega_1 t - k_1 x)$$
$$y_2(x,t) = a\sin(\omega_2 t - k_2 x)$$

The superposition of these two waves is given by

$$y = y_1 + y_2 = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

Using the trigonometric relation

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

we write the above equation as

$$y = y_1 + y_2 = 2a\sin\left[\frac{(\omega_1+\omega_2)}{2}t - \frac{(k_1+k_2)}{2}x\right]\cos\left[\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right] =$$
$$= 2a\sin[\omega t - kx]\cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right]$$

where $\omega = (\omega_1+\omega_2)/2$, $k = (k_1+k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$.

(ii) The resultant equation shows beats.

In the Figure two waves with slightly different frequencies are travelling to the right. Since the two waves are travelling in the same medium, they travel with the same speed. The resulting superposition sum wave travels in the same direction and with the same speed as the two component waves, but its local amplitude depends on whether the two individual waves have the same or opposite phase. The "beat" wave oscillates with the average frequency, and its amplitude envelope varies according to the difference frequency.

The resultant wave is seen to have the following two parts:

- A wave of angular frequency ω and propagation constant k, moving with a velocity

$$v_p = \frac{\omega}{k} = \nu\lambda$$

- A second wave of angular frequency $\frac{\Delta\omega}{2}$ and propagation constant $\Delta k/2$, moving with a velocity

$$v_g = \frac{\Delta\omega}{\Delta k}$$

When a number of plane waves of slightly different wavelengths travel in the same direction, they form wave groups or wave packets. The velocity with which the wave group advances in the medium is known as the group velocity υ_g. Each component wave has its own phase velocity, $\upsilon_p = \nu\lambda$. The wave packet has amplitude that is large in a small region and very small outside it. Such a variation of amplitude is called the modulation of the wave. The velocity of propagation of the modulation is known as the group velocity, υ_g.

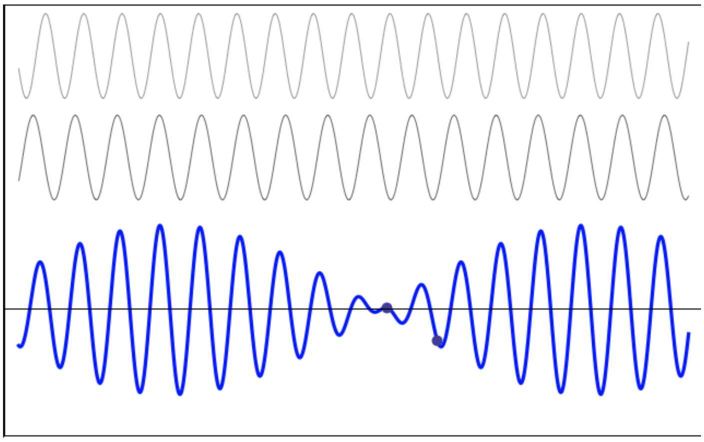

Fig. Beats are formed when two waves of slightly different frequencies combine
(a) The individual waves; (b) the resultant wave.

Answer: (i) $y = 2a\sin[\omega t - kx]\cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta kx}{2}\right]$ where $\omega = (\omega_1 + \omega_2)/2$, $k = (k_1 + k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$.