Answer on Question #74735 Physics / Other

A harmonic wave on a rope is described by $y(x,t) = 4.0 \sin\frac{2\pi}{0.82}(10t + x)$ mm.

i) Calculate the wavelength and time period of the wave.

ii) Determine the displacement and acceleration of the element of the rope located at $x = 0.58$ m at time, $t = 0.41$ s.

Solution:

The general equation of the harmonic wave is as follow

$$y(x,t) = A\sin(\omega t + kx)$$

where A is an amplitude of the displacement, ω is the angular frequency, k is a wave number.

In our case

$$A = 4 \text{ mm} = 0.004 \text{ m}$$

$$\omega = \frac{20\pi}{0.82}\ \frac{\text{rad}}{\text{s}}$$

$$k = \frac{2\pi}{0.82}\ \frac{1}{\text{m}}$$

i) The wavelength

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2\pi}{0.82}} = 0.82 \text{ m}$$

The time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{20\pi}{0.82}} = \frac{0.82}{10} = 0.082 \text{ s} = 82 \text{ ms}$$

ii) When $x = 0.58$ m and $t = 0.41$ s, we get

The displacement

$$y(0.58, 0.41) = 4.0 \sin\frac{2\pi}{0.82}(10 \times 0.41 + 0.58) \text{ mm} = -3.86 \text{ mm}$$

The acceleration

$$a(x,t) = \frac{\partial^2 y}{\partial t^2} = -4.0 \times \left(\frac{20\pi}{0.82}\right)^2 \sin\frac{2\pi}{0.82}(10t + x)\ \text{mm/s}^2$$

$$a(0.58, 0.41) = -4.0 \times \left(\frac{20\pi}{0.82}\right)^2 \sin\frac{2\pi}{0.82}(10 \times 0.41 + 0.58)\ \frac{\text{mm}}{\text{s}^2} = 22.65\ \frac{\text{m}}{\text{s}^2}$$

Answers: 0.82 m, 82 ms, -3.86 mm, $22.65\ \frac{\text{m}}{\text{s}^2}$