A particle is located upon by a force $\mathbf{F} = k(x^2 + x)\mathbf{i}$. What is the potential energy of the particle if it was zero at $x = 0$.

Solution

The relationship between a force and potential energy is as follows

$$\mathbf{F} = -\operatorname{grad} V(\mathbf{r}).$$

Therefore

$$V(\mathbf{r}) = -\int \mathbf{F} d\mathbf{r} = -\int k(x^2 + x)dx + C = -k\left(\frac{x^3}{3} + \frac{x^2}{2}\right) + C.$$

Because $V(0) = 0$ so $C = 0$.

Finally, the potential energy of the particle

$$V(x) = -k\left(\frac{x^3}{3} + \frac{x^2}{2}\right).$$

Answers: $V(x) = -k\left(\frac{x^3}{3} + \frac{x^2}{2}\right).$