Answer on Question #60744, Physics / Other |

Calculate the expectation value of kinetic energy of a simple harmonic oscillator in its ground state.

Solution:

Operator methods are very useful both for solving the Harmonic Oscillator problem and for any type of computation for the HO potential.

The Hamiltonian for the 1D Harmonic Oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

looks like it could be written as the square of a operator. It can be rewritten in terms of the operator A

$$A \equiv \left(\sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2m\hbar\omega}}\right)$$

and its Hermitian conjugate $A^\dagger$.

$$H = \hbar\omega\left(A^\dagger A + \frac{1}{2}\right)$$

The commutators with the Hamiltonian are easily computed.

$$\begin{aligned}[H, A] &= -\hbar\omega A \\ [H, A^\dagger] &= \hbar\omega A^\dagger\end{aligned}$$

From these commutators we can show that $A^\dagger$ is a raising operator for Harmonic Oscillator states

$$A^\dagger u_n = \sqrt{n+1}u_{n+1}$$

and that A is a lowering operator.

Because the lowering must stop at a ground state with positive energy, we can show that the allowed energies are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega.$$

In ground state

$$E_0 = \frac{1}{2}\hbar\omega$$

The expectation value of kinetic energy is

$$\begin{aligned}\langle u_n|\frac{p^2}{2m}|u_n\rangle &= \frac{-1}{2m}\frac{m\hbar\omega}{2}\langle u_n| - AA^\dagger - A^\dagger A|u_n\rangle \\ &= \frac{\hbar\omega}{4}\langle u_n|AA^\dagger + A^\dagger A|u_n\rangle \\ &= \frac{\hbar\omega}{4}((n+1)+n) = \frac{1}{2}E_n\end{aligned}$$

Thus, the expectation value of kinetic energy of a simple harmonic oscillator in its ground state

$$\langle T \rangle = \frac{1}{4}\hbar\omega$$

Answer: $\langle T \rangle = \frac{1}{4}\hbar\omega.$

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