Question #58439, Physics / Mechanics | Relativity

The radius of Earth is four times greater and its mass is 71 times bigger than that of the Moon. Find the length of the seconds pendulum near the surface of the moon.

Solution:

A **seconds pendulum** is a pendulum whose period is precisely two seconds: $T = 2\ s$.

$T = 2\pi\sqrt{\frac{l}{g}}$ - is a period of pendulum, where l – its length and g – gravitational acceleration on the surface on celestial body.

$g_E = 9.81\ m/s^2$– gravitational acceleration on the Earth surface, g_m – gravitational acceleration on the Moon surface.

$g = G\frac{M}{R^2}$, where $G = 6{,}672 \times 10^{-11}\frac{M^2}{kg\cdot s^2}$ is the gravitational constant, M and R – mass and radius of celestial ,body.

With the conditions of problem $\frac{M_E}{M_M} = 71$ and $\frac{R_E}{R_M} = 4$.

$T = 2\pi\sqrt{\frac{l_M}{g_M}}$ ➔ l_M–?

Define l_M:

$$\sqrt{\frac{l_M}{g_M}} = \frac{T}{2\pi} \Rightarrow \frac{l_M}{g_M} = \left(\frac{T}{2\pi}\right)^2 \Rightarrow l_M = g_M\left(\frac{T}{2\pi}\right)^2;$$

and

$$\frac{g_M}{g_E} = \frac{G\frac{M_M}{R_M^2}}{G\frac{M_E}{R_E^2}} = \frac{M_M\cdot R_E^2}{M_E\cdot R_M^2},\ M_E = 71M_M \text{ and } R_E = 4R\_M \Rightarrow \frac{g_M}{g_E} = \frac{M_M\cdot(4R_M)^2}{71M\_E\cdot R_M^2} = \frac{16\cdot M_M\cdot R_M^2}{71\cdot M_M\cdot R_M^2} = \frac{16}{71}$$

$$g_M = \frac{16}{71}\cdot g_E$$

$$l_M = g_M\left(\frac{T}{2\pi}\right)^2 = \frac{16}{71}\cdot g_E\cdot\left(\frac{T}{2\pi}\right)^2 = \frac{16}{71}\cdot 9.81\ m/s^2\left(\frac{2s}{2\pi}\right)^2 = \frac{16}{71}\cdot 9.81\cdot\left(\frac{1}{\pi}\right)^2[m] = 0.22\ m = 22\ cm.$$

$$l_m = \frac{16g_E}{71\ \pi^2}.$$

Answer: The length of the seconds pendulum near the surface of the moon is equal to 22 cm.