Answer on Question #54056, Physics Mechanics Kinematics Dynamics

a hollow sphere and a solid sphere, both having the same mass of 5kg and radius 10m are initially at rest. if they are made to roll down on the same plane without slipping, the ratio of their speeds when they reach the bottom of the plane, vhollow/vsolid will be

Solution

According to the law of energy conservation for the hollow sphere

$$mgh = \frac{mv_{hollow}^2}{2} + \frac{J_{hollow}\omega_{hollow}^2}{2} \quad (1)$$

where $v_{hollow} = r\omega_{hollow}$; $J_{hollow} = \frac{2}{3}mr^2$ is the moment of inertia of a hollow sphere.

Then

$$mgh = \frac{mv_{hollow}^2}{2} + \frac{\frac{2}{3}mr^2\omega_{hollow}^2}{2} = \frac{5mv_{hollow}^2}{6} \Rightarrow v_{hollow} = \sqrt{\frac{6gh}{5}}$$

According to the law of energy conservation for the solid sphere

$$mgh = \frac{mv_{solid}^2}{2} + \frac{J_{solid}\omega_{solid}^2}{2} \quad (2)$$

where $v_{solid} = r\omega_{solid}$; $J_{solid} = \frac{2}{5}mr^2$ is the moment of inertia of a hollow sphere.

Then

$$mgh = \frac{mv_{solid}^2}{2} + \frac{\frac{2}{5}mr^2\omega_{solid}^2}{2} = \frac{7mv_{solid}^2}{10} \Rightarrow v_{solid} = \sqrt{\frac{10gh}{7}}$$

So, $v_{hollow} / v_{solid} = \sqrt{\frac{6gh}{5}} / \sqrt{\frac{10gh}{7}} = \sqrt{\frac{42}{50}} = \frac{\sqrt{21}}{5} \approx 0.917$

Answer: $v_{hollow} / v_{solid} = \frac{\sqrt{21}}{5} \approx 0.917$