Answer on Question#53117 – Physics / Mechanics – Kinematics – Dynamics

$4.905\ kJ$

Question

A cubicle vessel of height 1 m is full of water. What will be the minimum work done in taking water out from vessel?

Solution

Consider an infinitely small volume dV of water (elementary volume).

Minimum work to take it out from vessel – transfer to the highest point of the vessel ($dW = g(h - h_i)dm$) and shift horizontally out of vessel (assume that there is no friction with medium (e.g air), so no contribution to work there).

$g -$ gravitational acceleration, $h -$ height of vessel, $h_i -$ initial height of elementary volume, $dm -$ mass.

Thus, our aim to calculate work over all heights h_i. Assume that the lowest point has $h_i = 0\ m$, accordingly to the data, the highest point has $h_i = 1\ m$ then (Note: no effect on result). At the same time (in general) we can rewrite $dm = \frac{dm}{dh}dh$, and due to assumption that density of water independent from height, we can replace $\frac{dm}{dh}$ with $\frac{m}{h}$. Finally, we can jot down proper integral:

$$W = \int_{h_{imin}}^{h_{imax}} g(h - h_i)\frac{m}{h}\ dh_i$$

Let us put in actual boundaries and solve it:

$$W = \int_0^1 g(h - h_i)\frac{m}{h}\ dh_i = \frac{mg}{h}\int_0^1 (h - h_i)dh_i = mg\int_0^1 dh_i\ - \frac{mg}{h}\int_0^1 h_i dh_i\ =$$

$$= mgh_i\Big|_0^1 - \frac{mg}{h}\frac{h_i^2}{2}\Big|_0^1 = mg - \frac{mg}{2h} = mg\left(1 - \frac{1}{2h}\right)$$

There are few numbers from tables we required to know to get numeric answer: (mass of $1\ m^3$ of water) $m \approx 1000\ kg$, (gravitational acceleration) $g \approx 9.81\ ms^{-2}$. Substitute them:

$$W = mg\left(1 - \frac{1}{2h}\right) = 1000 * 9.81\left(1 - \frac{1}{2 * 1}\right) = 4905\ J = 4.905\ kJ$$