Answer on Question #51238-Physics-Solid State Physics

Derive an expression for the velocity of the transverse wave in the [100] direction in a cubic crystal.

Solution

The equation of motion in the x direction is

$$\rho\frac{\partial^2 u}{\partial t^2}=C_{11}\frac{\partial^2 u}{\partial x^2}+C_{44}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+(C_{12}+C_{44})\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right).\quad (a)$$

The equation of motion in the y direction is

$$\rho\frac{\partial^2 v}{\partial t^2}=C_{11}\frac{\partial^2 v}{\partial x^2}+C_{44}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+(C_{12}+C_{44})\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}\right).\quad (b)$$

The equation of motion in the z direction is

$$\rho\frac{\partial^2 w}{\partial t^2}=C_{11}\frac{\partial^2 w}{\partial x^2}+C_{44}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+(C_{12}+C_{44})\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial y\partial z}\right).\quad (c)$$

here ρ is the density and u, v, w are the components of the displacement, C_{ab} are the elastic stiffness constants.

Consider a transverse or shear wave with the wavevector along the x cube edge and with the particle displacement v in the y direction:

$$v=v_0e^{i(Kx-\omega t)}.$$

On substitution in (b) this gives the dispersion relation

$$\omega^2\rho=C_{44}K^2;$$

thus the velocity $\frac{\omega}{K}$ of a transverse wave in the [100] direction is

$$v_s=\left(\frac{C_{44}}{\rho}\right)^2.$$

The identical velocity is obtained if the particle displacement is in the z direction.

Thus for K parallel to [100] the two independent shear waves have equal velocities. This is not true for K in a general direction in the crystal.