Answer on Question #47580 – Physics – Quantum Mechanics

Question.

Calculate the first-order energy corrections for all levels of an infinite square well potential (between x = 0 and x = L) with a perturbation H' = αx(L-x).

$H' = \alpha x(L - x)$

$E_n^{(1)} = ?$

Solution.

Pesturbation theory helps to find an approximate solution of a problem. From the pesturbation theory we know that the first-order energy corrections is the following:

$$E_n^{(1)} = H'_{nn} = \int_{-\infty}^{+\infty} \bar{\psi}(x) H' \psi(x) dx$$

$\psi(x)$ is the wave function;

H' is the pesturbation.

In the case of an infinite square well potential the wave functions is equal to:

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right); n \in Z$$

So, we can calculate the first-order energy corrections:

$$E_n^{(1)} = H'_{nn} = \int_{-\infty}^{+\infty} \bar{\psi}(x) H' \psi(x) dx = \frac{2\alpha}{L} \int_0^L \sin^2\left(\frac{\pi n x}{L}\right) x(L - x) dx =$$
$$= \frac{2\alpha}{L} \int_0^L \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n x}{L}\right)\right)(xL - x^2) dx =$$
$$= \frac{\alpha}{L} \int_0^L xL - x^2 - xL\cos\left(\frac{2\pi n x}{L}\right) + x^2 \cos\left(\frac{2\pi n x}{L}\right) dx$$

Thus, we received the sum of four integrals. Let calculate each of them.

$$\int_0^L xL dx = \frac{L^3}{2};$$

$$\int_0^L x^2 dx = \frac{L^3}{3};$$

$$\int_0^L x\cos\left(\frac{2\pi n x}{L}\right) dx = 0$$

$$\int_0^L x^2 \cos\left(\frac{2\pi n x}{L}\right) dx = \frac{L^3}{(2\pi n)^2}$$

Finally,

$$E_n^{(1)} = \frac{\alpha}{L}\int_0^L xL - x^2 - xL\cos\left(\frac{2\pi n x}{L}\right) + x^2\cos\left(\frac{2\pi n x}{L}\right) dx = \frac{\alpha}{L}\left(\frac{L^3}{2} - \frac{L^3}{3} + \frac{L^3}{(2\pi n)^2}\right) =$$

$$= \alpha L^2\left(\frac{1}{6} + \frac{1}{(2\pi n)^2}\right)$$

Answer.

$$E_n^{(1)} = \alpha L^2\left(\frac{1}{6} + \frac{1}{(2\pi n)^2}\right)$$