Question #28752

Establish the differential equation for a damped oscillator. Show that, for weak damping, the solution of the differential equation for the damped oscillator is given by
x(t) = a0 exp (−bt) cos (wd t + f)

To incorporate friction, we can just say that there is a frictional force that's proportional to the velocity of the mass. This is a pretty good approximation for a body moving at a low velocity in air, or in a liquid. So we say the frictional force $f_r = -bv$. The constant b depends on the kind of liquid the mass is in and the shape of the mass. The negative sign, just says that the force is in the opposite direction to the body's motion. Let's add this frictional force in to the equation $f_{net} = ma$

$$-kx - bv = ma \tag{1.59}$$

In terms of derivatives

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{1.60}$$

This is a differential equation. We'll solve it using the guess method.

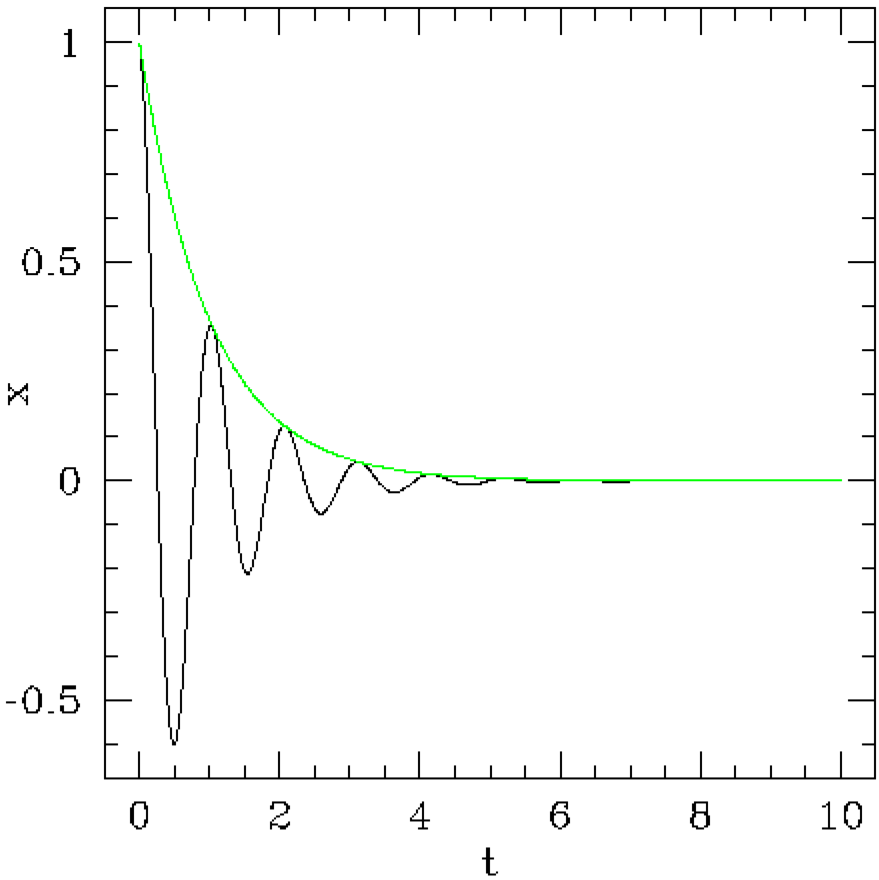
We will predict that instead of the amplitude being constant, it's decaying with time.

$$A(t) = A_0 e^{-const\ t} \tag{1.61}$$

So

$$x(t) = A(t)\cos(\omega t + \delta) = A_0 e^{-const\ t}\cos(\omega t + \delta) \tag{1.62}$$

Here's a plot of of an example of such a function $x(t) = e^{-t}\cos(2\pi t)$



The green line is $A(t) = e^{-t}$. It is the envelope of the oscillation. Obviously depending on the rate of decay of the amplitude, and the frequency, you'll get a different picture. But qualitatively you'll see an oscillating function whose amplitude decays away to zero. This should describe weak damping. We don't expect this to work too well in molasses. To get a more quantitative understanding we'll have to do some more math.

We'll try sticking $x(t) = Ae^{\lambda t}$ into eqn. 1.60. Here again, A is just a constant. We already differentiated this function before in eqns. 1.24 and 1.25 so we don't have to do it again. So we have

$$m\lambda^2 x + b\lambda x + kx = 0 \tag{1.63}$$

Canceling the x's

$$m\lambda^2 + b\lambda + k = 0 \tag{1.64}$$

This is a quadratic equation for λ. Let's solve it:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} \tag{1.65}$$

So we have two possible solutions for λ! They both solve the equation, and we have to have more information to figure out what to do with them. But for the moment, let's look at this equation more closely.

If the damping, b, is large, then the square root is real. However if $b^2 < 4mk$, then it becomes imaginary.

Source

Please refer to http://physics.ucsc.edu/~josh/6A/book/harmonic/node18.html for the original and more information.