Question 13297

a) $x = a\sin\omega t$, so velocity $v = \dot{x} = a\omega\cos\omega t$. One knows, that the parametric equation of the ellipse in coordinates (x, y) is $x = A\sin\omega t, y = B\cos\omega t$. Comparing it with equations for x, v , in (x, v) they will represent an ellipse with $A = a, B = a\omega$

b) Let the energy $E = \frac{m\dot{x}^2}{2} + \frac{k x^2}{2}$ be constant. One might then rewrite the last equation as $\frac{m\dot{x}^2}{2E} + \frac{k x^2}{2E} = 1$, or $\frac{\dot{x}^2}{\frac{2E}{m}} + \frac{x^2}{\frac{2E}{k}} = 1$. The equation of ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. So, constant energy represents the ellipse with $a = \sqrt{2\frac{E}{k}}, b = \sqrt{2\frac{E}{m}}$, which are constant when $E = const$.