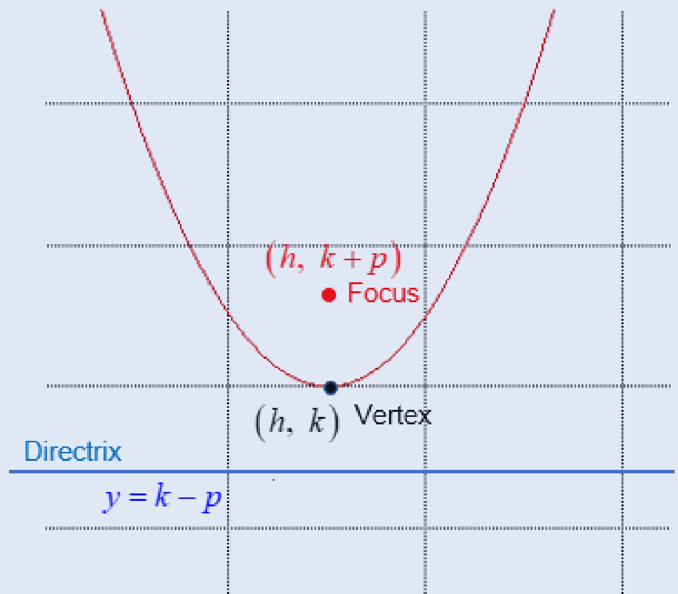
Case-I

For a parabola of the shape as shown beside



The standard equation is

$$(x-h)^2 = 4p(y-k)$$

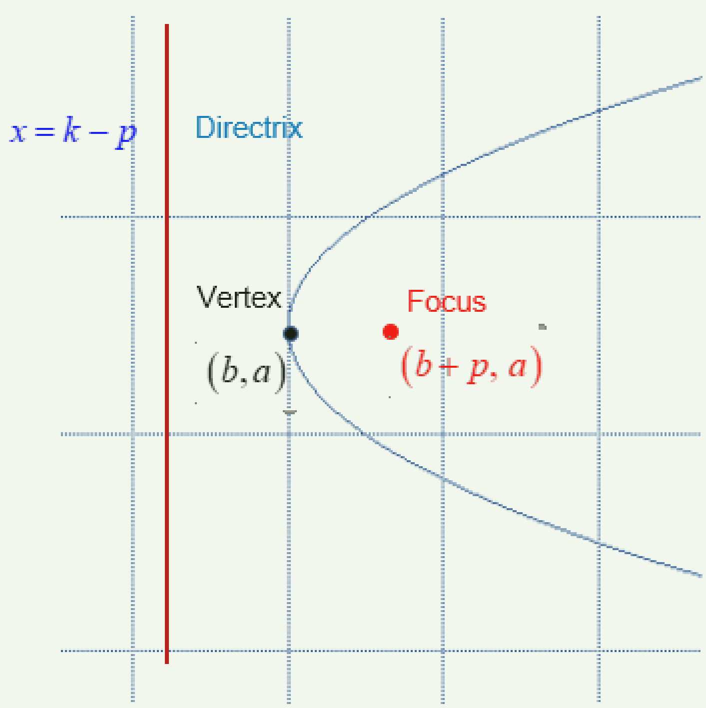
Vertex is $(h,\ k)$

Focus is $(h,\ k+p)$

And the equation of directrix is
$y = k - p$

Case-II

For a parabola of the shape shown beside



A standard equation of Parabola is

$$(y-a)^2 = 4p(x-b)$$

Vertex is $(b,\ a)$

Focus is $(b+p,\ a)$

And the equation of directrix is

$x = b - p$

(1). For the parabola, $(x+2)^2 = 20(y+5)$

Find the vertex, focus, and Directrix of each parabola

Solution

The given equation of the parabola is

$$(x+2)^2 = 20(y+5) \qquad \ldots \qquad (1)$$

This equation can be re-written as:

$$[x-(-2)]^2 = 4\times 5[y-(-5)] \qquad \ldots \qquad (2)$$

Comparing equation (2) with equation of parabola, $(x-h)^2 = 4p(y-k)$,

We have, $h=-2$, $k=-5$, $p=5$, therefore,

Vertex

$(h,\ k) = (-2,-5)$

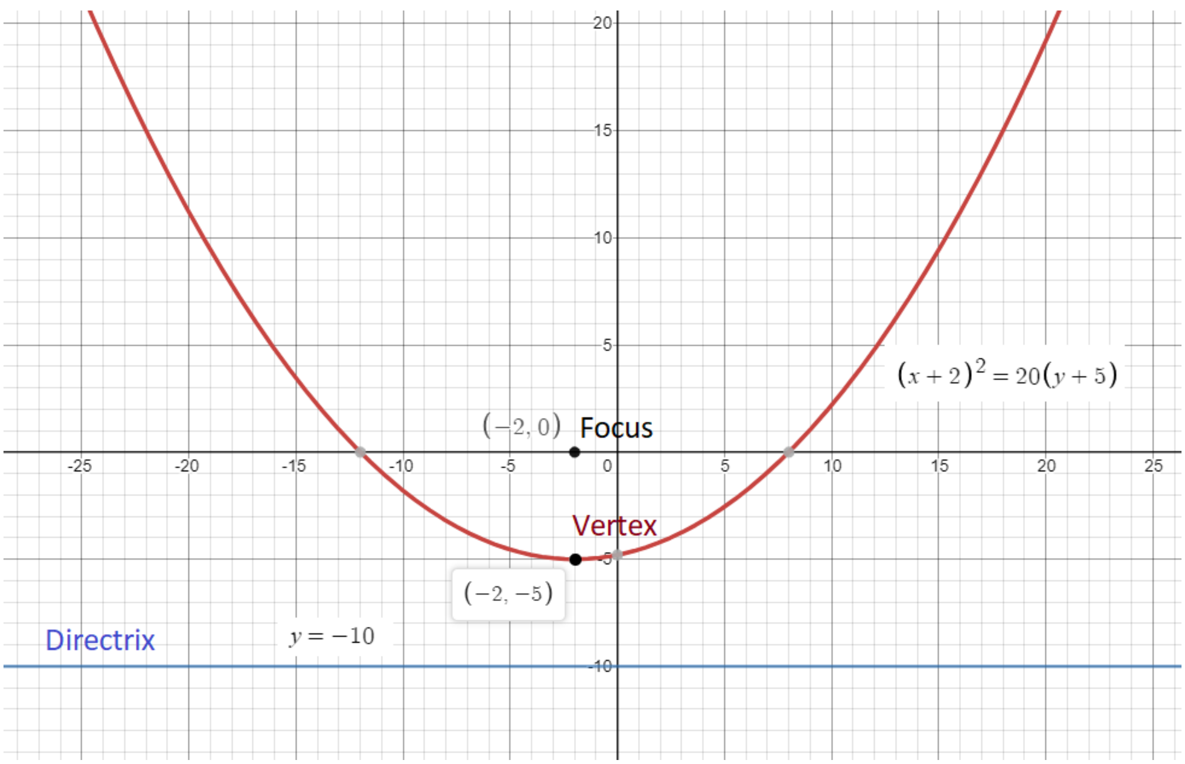
Focus is

$(h,\ k+p) = (-2,-5+5)$

$= (-2,0)$

And equation of directrix is $y = k - p = -5-5 \quad \Rightarrow \quad y=-10$

The given parabola, with its vertex, focus and the directrix is shown below



(3). For the parabola, $(x-5)^2 = 4(y+1)$

Find the vertex, focus, and Directrix of each parabola

Solution

The given equation of the parabola is

$$(x-5)^2 = 4(y+1) \qquad \ldots \qquad (1)$$

This equation can be re-written as:

$$[x-(5)]^2 = 4\times 1[y-(-1)] \qquad \ldots \qquad (2)$$

Comparing equation (2) with equation of parabola, $(x-h)^2 = 4p(y-k)$,

We have, $h=5$, $k=-1$, $p=1$, therefore

Vertex

$(h,\ k) = (5,-1)$

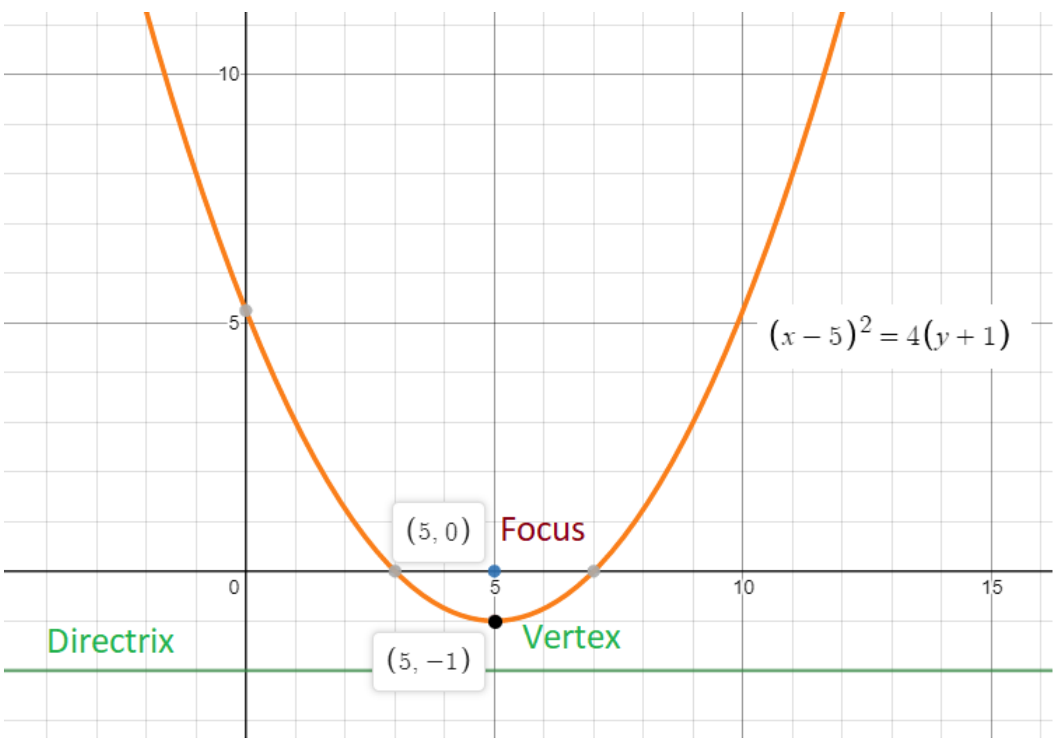
Focus is

$(h,\ k+p) = (5,-1+1)$

$= (5,0)$

And equation of directrix is $y = k - p = -1-1 \quad \Rightarrow \quad y=-2$

The given parabola, with its vertex, focus and the directrix is shown below



(5). For the parabola, $(x+4)^2 = 6(y-5)$

Find the vertex, focus, and Directrix of each parabola

Solution

The given equation of the parabola is

$$(x+4)^2 = 6(y-5) \qquad \ldots \qquad (1)$$

This equation can be re-written as:

$$[x-(-4)]^2 = 4\times\frac{3}{2}[y-(5)] \qquad \ldots \qquad (2)$$

Comparing equation (2) with equation of parabola, $(x-h)^2 = 4p(y-k)$,

We have, $h=-4$, $k=5$, $p=\frac{3}{2}$, therefore

Vertex

$(h,\ k) = (-4, 5)$

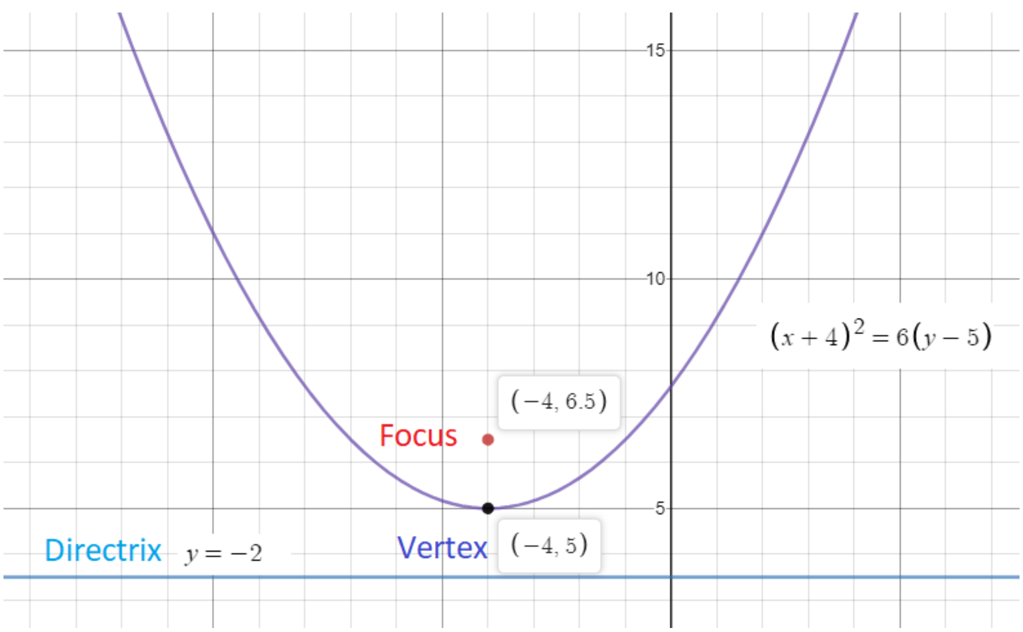
Focus is

$(h,\ k+p) = \left(-4, 5+\frac{3}{2}\right) = \left(-4, \frac{13}{2}\right)$

And equation of directrix is

$$y = k-p = 5-\frac{3}{2} \qquad \Rightarrow \qquad y=\frac{7}{2}$$

The given parabola, with its vertex, focus and the directrix is shown below



(2). For the parabola, $(y-1)^2=-16(x+4)$

Find the vertex, focus, and Directrix of each parabola

Solution

The given equation of the parabola is

$$(y-1)^2=-16(x+4) \qquad \dots \qquad (1)$$

This equation can be re-written as:

$$[y-(1)]^2=4\times(-4)[x-(-4)] \qquad \dots \qquad (2)$$

Comparing equation (2) with equation of parabola, $(y-a)^2=4p(x-b)$,

We have, $b=-4$, $a=1$, $p=-4$, therefore

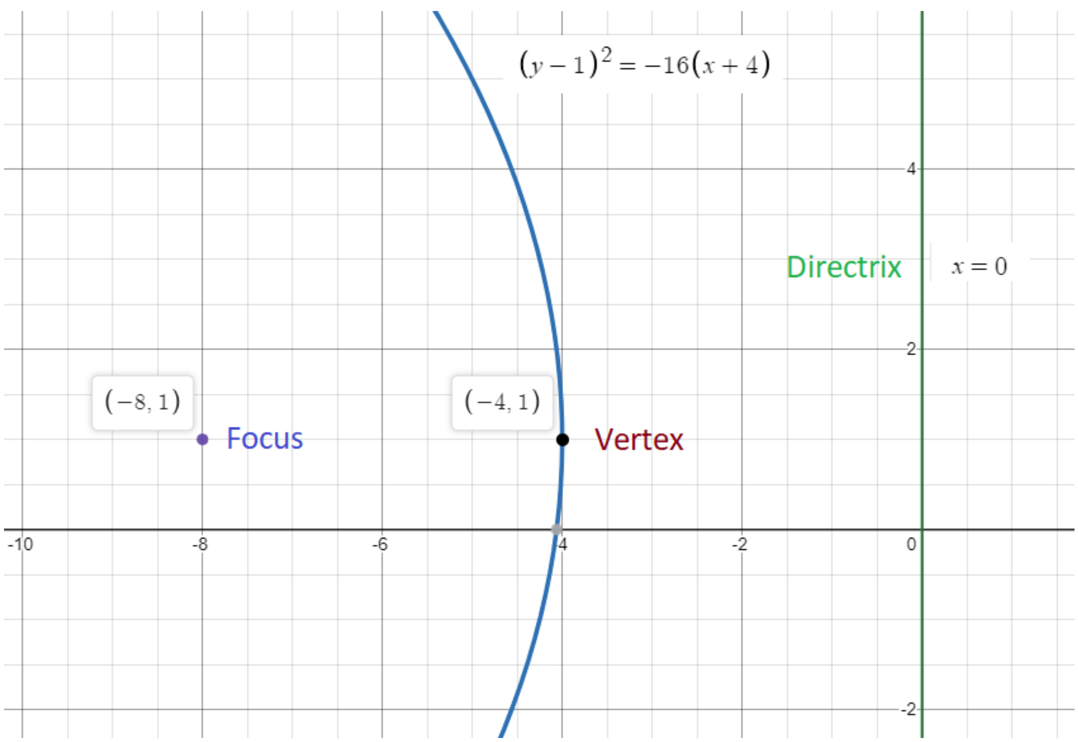
Vertex
$(b,\ a)=(-4,1)$

Focus is
$(b+p,\ a)=(-4-4,1)=(-8,1)$

And equation of directrix is

$$x=b-p=-4-(-4) \Rightarrow \qquad x=0$$

The given parabola, with its vertex, focus and the directrix is shown below



(4). For the parabola, $(y-3)^2 = 12(x+1)$

Find the vertex, focus, and Directrix of each parabola

Solution

The given equation of the parabola is

$$(y-3)^2 = 12(x+1) \qquad \ldots \qquad (1)$$

This equation can be re-written as:

$$[y-(3)]^2 = 4\times(3)[x-(-1)] \qquad \ldots \qquad (2)$$

Comparing equation (2) with equation of parabola, $(y-a)^2 = 4p(x-b)$,

We have, $b=-1$, $a=3$, $p=3$, therefore

Vertex
$(b,\ a) = (-1,\ 3)$

Focus is
$(b+p,\ a) = (-1+3, 3) = (2,\ 3)$

And equation of directrix is

$$x = b - p = -1-(3) \quad \Rightarrow \quad x = -4$$

The given parabola, with its vertex, focus and the directrix is shown below