Answer to the Question #89436 – Math – Calculus

Question

Determine whether Rolle's theorem can be applied to f on the close interval [a,b]. if it can be applied, find the values of c in open interval (a, b) such that f'(c)=0. f(x)=(x^2-2x-3)/(x+2), [-1,3]

Solution

$$f(x) = \frac{x^2 - 2x - 3}{x + 2}$$

$$f(x) = \frac{x^2 - 3x + x - 3}{x + 2}$$

$$f(x) = \frac{x(x - 3) + 1(x - 3)}{x + 2}$$

$$f(x) = \frac{(x - 3)(x + 1)}{x + 2}$$

$f(x)$ is defined for all values of x except -2.

Given interval is $[-1,3]$ and $x \notin [-1,3]$.

$\therefore f(x)$ is a rational function and continuous for $[-1,3]$.

Also, $f(x)$ is a differentiable function in (-1,3)

Now, $f(\text{a}) = f(b)$

$f(-1) = f(3)$

$$\frac{(-1-3)(-1+1)}{-1+2} = \frac{(3-3)(3+1)}{3+2}$$

0=0

It is true.

Then, there exists a $c \in$ (-1,3) such that $f'(c)$=0.

Now, $f'(x) = \dfrac{(x+2)(x^2-2x-3)' - (x^2-2x-3)(x+2)'}{(x+2)^2}$ [$\because$ ıotient's rule]

$$f'(x) = \frac{(x+2)(2x-2) - (x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+4x-2x-4-x^2+2x+3}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$$

$Now, f'(c) = 0$

$$\frac{c^2+4c-1}{(\text{c}+2)^2} = 0$$

$c^2+4c-1=0$

$$c = \frac{-4 \pm \sqrt{4^2-4(1)(-1)}}{2(1)}$$

$$c = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm \sqrt{20}}{2}$$

$$c = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$

$\because$ 3)

we take $c = -2 + \sqrt{5} \in (-1,3)$

Thus, $c = -2 + \sqrt{5}$.

Answer: $c = -2 + \sqrt{5}$.