Answer to Question #88847 – Math – Algebra

Question

Given that alpha and beta are the roots of the equation 2xsquare - 4x + 3 = 0. Form a new quadratic equation whose root are :

A. 1/alpha, 1/beta

B. 2alpha - 1/beta, 2 beta^-1/2

C. 2alpha, 2beta

Solution

$2x^2 - 4x + 3 = 0$

Let α, β be the roots.

then, $\alpha + \beta = -\frac{b}{a} = -\frac{-4}{2} = 2$

$\alpha\beta = \frac{c}{a} = \frac{3}{2}.$

A.

Let $\frac{1}{\alpha}, \frac{1}{\beta}$ be the roots of $px^2 + qx + r = 0$ or $x^2 + \frac{q}{p}x + \frac{r}{p} = 0$

then, $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{q}{p}$ and $\frac{1}{\alpha}\frac{1}{\beta} = \frac{r}{p}$

$\frac{\alpha + \beta}{\alpha\beta} = -\frac{q}{p}$ and $\frac{1}{\alpha\beta} = \frac{r}{p}$

$\frac{2}{3/2} = -\frac{q}{p}$ and $\frac{1}{3/2} = \frac{r}{p}$

$-\frac{4}{3} = \frac{q}{p}$ and $\frac{2}{3} = \frac{r}{p}$

$\therefore$ Equation is : $x^2 - \frac{4}{3}x + \frac{2}{3} = 0$

or, $3x^2 - 4x + 2 = 0.$

B.

$$\text{Let } (2\alpha - \frac{1}{\beta}) \text{ and } (2\beta - \frac{1}{\alpha}) \text{ be the roots of } px^2 + qx + r = 0 \text{ or } x^2 + \frac{q}{p}x + \frac{r}{p} = 0$$

$$\text{then}, 2\alpha - \frac{1}{\beta} + 2\beta - \frac{1}{\alpha} = -\frac{q}{p} \text{ and } (2\alpha - \frac{1}{\beta})(2\beta - \frac{1}{\alpha}) = \frac{r}{p}$$

$$2(\alpha + \beta) - (\frac{1}{\beta} + \frac{1}{\alpha}) = -\frac{q}{p} \text{ and } 4\alpha\beta - 2 - 2 + \frac{1}{\alpha\beta} = \frac{r}{p}$$

$$2(2) - (\frac{4}{3}) = -\frac{q}{p} \text{ and } 4(\frac{3}{2}) - 4 + \frac{1}{3/2} = \frac{r}{p}$$

$$4 - \frac{4}{3} = -\frac{q}{p} \text{ and } 6 - 4 + \frac{2}{3} = \frac{r}{p}$$

$$-\frac{8}{3} = \frac{q}{p} \text{ and } \frac{8}{3} = \frac{r}{p}$$

$$\therefore \text{Equation is}: x^2 - \frac{8}{3}x + \frac{8}{3} = 0$$

$$\text{or}, 3x^2 - 8x + 8 = 0.$$

C.

$$\text{Let } (2\alpha) \text{ and } (2\beta) \text{ be the roots of } px^2 + qx + r = 0 \text{ or } x^2 + \frac{q}{p}x + \frac{r}{p} = 0$$

$$\text{then}, 2\alpha + 2\beta = -\frac{q}{p} \text{ and } (2\alpha)(2\beta) = \frac{r}{p}$$

$$2(\alpha + \beta) = -\frac{q}{p} \text{ and } 4\alpha\beta = \frac{r}{p}$$

$$2(2) = -\frac{q}{p} \text{ and } 4(\frac{3}{2}) = \frac{r}{p}$$

$$4 = -\frac{q}{p} \text{ and } 6 = \frac{r}{p}$$

$$-4 = \frac{q}{p} \text{ and } 6 = \frac{r}{p}$$

$$\therefore \text{Equation is}: x^2 - 4x + 6 = 0$$

or, $x^2 - 4x + 6 = 0$.