Answer to Question #88759 – Math – Differential Equations

Question

Find the particular solution of dy/dx+2y=2x^2+3

Solution

$$\frac{dy}{dx} + 2y = 2x^2 + 3 \ldots \text{(i)}$$

On comparing with,

$$\frac{dy}{dx} + Py = Q$$

We get,

$$P = 2, Q = 2x^2 + 3$$

Then, Integrating factor $(IF) = e^{\int P dx} = e^{\int 2dx} = e^{2x}$

Now multiplying both sides of (i) with IF, we get

$$y.IF = \int (2x^2 + 3) IF \, dx$$

$$ye^{2x} = \int (2x^2 + 3) e^{2x} \, dx$$

Apply integrating by parts to right side,

$$ye^{2x} = (2x^2 + 3)\int e^{2x} \, dx - \int \{\frac{d}{dx}(2x^2 + 3)\int e^{2x} dx\} dx$$

$$ye^{2x} = (2x^2 + 3)\frac{e^{2x}}{2} - \int \{4x \square \frac{e^{2x}}{2}$$

$$ye^{2x} = (2x^2 + 3)\frac{e^{2x}}{2} - \int \{2xe^{2x}\} dx$$

Again Apply integrating by parts,

$$ye^{2x} = (2x^2 + 3)\frac{e^{2x}}{2} - [2x\int e^{2x} dx - \int 2 \square \frac{e^{2x}}{2}$$

$$ye^{2x} = (2x^2 + 3)\frac{e^{2x}}{2} - [2x \square \frac{e^{2x}}{2} \quad \frac{e^{2x}}{2}] + C \quad [\text{where C is constant of integration}]$$

$$ye^{2x} = (2x^2 + 3)\frac{e^{2x}}{2} - xe^{2x} + \frac{e^{2x}}{2} + C$$

$$y = \frac{2x^2 + 3}{2} - x + \frac{1}{2} + Ce^{-2x}.$$