Answer to Question #88628 – Math – Calculus

Question

A solid in the shape of a hemisphere with a radius of 2 units, has its base in the xy-plane and the centre of the base at the origin. If the density of the solid is given by the function ρ(x, y, z) = xyz, determine the mass of the hemisphere.

Solution

Mass of the hemisphere = $\int \rho(x,y,z)dV$

Given, $\rho(x,y,z) = xyz$. Changing to spherical coordinates,

$x = rsin\theta cos\varphi$; $y = rsin\theta sin\varphi$; $z = rcos\theta$.

This gives,

$$Mass\ of\ the\ hemisphere = \int \rho(x,y,z)dV$$

$$= \int_0^2 \int_0^{2\pi} \int_0^{\pi/2} (rsin\theta cos\varphi)(rsin\theta sin\varphi)(rcos\theta)r^2 \sin\theta\, d\theta d\varphi dr$$

$$= \int_0^2 r^5 dr \int_0^{2\pi} cos\varphi sin\varphi d\varphi \int_0^{\frac{\pi}{2}} \sin^3\theta cos\theta d\theta$$

$$= \left(\frac{2^6}{6}\right) * \frac{1}{2}(\sin^2 2\pi - \sin^2 0) * \frac{1}{2} * \frac{1}{2} = 0$$

Hence, the mass of the hemisphere is zero.