Answer to Question #88535 – Math – Trigonometry

Question

If sinØ=3/5 and Ø is acute find sin½Ø

Solution

$\sin \emptyset = \frac{3}{5}$

We know that $sin^2\emptyset + cos^2\emptyset = 1 \quad \Rightarrow cos^2\emptyset = 1 - sin^2\emptyset$

$\cos \emptyset = \sqrt{1 - sin^2\emptyset}$ (By taking square root on both sides)

$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$ (by substituting $\sin \emptyset = \frac{3}{5}$)

$= \sqrt{1 - \frac{9}{25}}$

$= \sqrt{\frac{25-9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \sqrt{\left(\frac{4}{5}\right)^2}$

$= \frac{4}{5}$ (Since $\sqrt{x^2} = x$)

Therefore, $\cos \emptyset = \frac{4}{5}$.

We know that $\emptyset$ is an acute angle, then $\emptyset/2$ also will be an acute angle. Besides,

$\cos \emptyset = 1 - 2sin^2\left(\frac{\emptyset}{2}\right)$

$\Rightarrow 2sin^2\left(\frac{\emptyset}{2}\right) = 1 - \cos \emptyset$

$\Rightarrow sin^2\left(\frac{\emptyset}{2}\right) = \frac{1-\cos \emptyset}{2}$ (by dividing both sides by 2)

$\Rightarrow sin\left(\frac{\emptyset}{2}\right) = \sqrt{\frac{1-\cos \emptyset}{2}}$ (by taking square root on both sides)

$= \sqrt{\frac{1-\frac{4}{5}}{2}}$ (by substituting $\cos \emptyset = \frac{4}{5}$)

$= \sqrt{\frac{\frac{5-4}{5}}{2}}$

$= \sqrt{\frac{\left(\frac{1}{5}\right)}{2}} = \sqrt{\frac{1}{10}}$

Therefore, $sin\left(\frac{\emptyset}{2}\right) = \frac{1}{\sqrt{10}} = 0.3162.$