## Question #88320, Math, Differential Equations

1. If $f$ and $g$ are arbitrary functions of their respective arguments, show that $u = f(x - vt + iay) + g(x - vt + iay)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, where $a^2 = 1 - \frac{v^2}{c^2}$

**Solution:**

Consider $u = f(x - vt + iay) + g(x - vt + iay)$.

Differentiating $u$ partially with respect to $x$, we get

$$\begin{aligned} \frac{\partial u}{\partial x} &= f'(x - vt + iay) + g'(x - vt + iay) \\ \frac{\partial^2 u}{\partial x^2} &= f''(x - vt + iay) + g''(x - vt + iay) \qquad (1) \end{aligned}$$

Differentiating $u$ partially with respect to $y$, we get

$$\begin{aligned} \frac{\partial u}{\partial y} &= f'(x - vt + iay) \cdot ia + g'(x - vt + iay) \cdot ia \\ \frac{\partial^2 u}{\partial y^2} &= f''(x - vt + iay) \cdot (-a^2) + g''(x - vt + iay) \cdot (-a^2) \\ &= -a^2 \left(\frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(by using Equation 1)} \qquad (2) \end{aligned}$$

Differentiating $u$ partially with respect to $t$, we get

$$\begin{aligned} \frac{\partial u}{\partial t} &= f'(x - vt + iay) \cdot (-v) + g'(x - vt + iay) \cdot (-v) \\ \frac{\partial^2 u}{\partial t^2} &= f''(x - vt + iay) \cdot (v^2) + g''(x - vt + iay) \cdot (v^2) \\ &= v^2 \left(\frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(by using Equation 1)} \qquad (3) \end{aligned}$$

Taking $a^2 = 1 - \frac{v^2}{c^2}$, Equation (2) becomes,

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= -a^2 \left(\frac{\partial^2 u}{\partial x^2}\right) \\ &= -\left(1 - \frac{v^2}{c^2}\right)\left(\frac{\partial^2 u}{\partial x^2}\right) \\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= \frac{v^2}{c^2}\left(\frac{\partial^2 u}{\partial x^2}\right) \\ &= \frac{1}{c^2}\left(v^2\left(\frac{\partial^2 u}{\partial x^2}\right)\right) \\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad \text{(by using Equation 3)} \qquad (4) \end{aligned}$$

From Equation (4), $u = f(x - vt + iay) + g(x - vt + iay)$ satisfies the given differential equation and hence the solution of the equation.

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